

Chair of Experimental Solid State Physics, LMU Munich

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“Introduction to Graphene  
and 2D Materials”

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SS24 Lecture 1, 15/4/2024

# Lecture Structure

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## Summer term SS 2024

Event	Room	Time	Lecturer
Lecture "Introduction to Graphene and 2D Materials"  Lecture Materials:  <a href="#">Lecture outline</a>  <a href="#">Lecture 1</a>	Geschw.-Scholl-Pl. 1 (N)/Kleiner Physiksaal (N 020)	Mon. 8:30am – 10:00am  Begin: 15.04.24  End: 15.07.24	Prof. Dr. D.K.Efetov
Tutorials "Introduction to Graphene and 2D Materials"  Exercises:	Geschw.-Scholl-Pl. 1 (N)/Kleiner Physiksaal (N 020)	Fri. 08:30am – 10:00am  Fri. 10:30am – 12:00pm  Begin: 19.04.24  End: 19.07.24	Dr.M.Lee

→ More info on <https://www.quantummatter.physik.lmu.de/>

# Lecture Structure

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## **Lecture and Tutorials of “Introduction to Graphene and 2D Materials”**

This class builds upon the “E\_M1 Advanced Solid State Physics” lecture and develops an introductory-level insight into the main concepts and the rich phenomenology of graphene and other two-dimensional materials, leading up to the recent advancements in moiré superlattices. In particular, the class aims to introduce all the main concepts and techniques that are needed for the study of the key experimental literature on the emergent field of moiré materials, with a strong bias towards low-temperature electronic experiments.

### **Lecture:**

Lecturer: Prof. Dr. Dmitri K. Efetov, E-mail: [dmitri.efetov@lmu.de](mailto:dmitri.efetov@lmu.de)  
Mon. 8:30am – 10:00am, Geschw.-Scholl-Pl. 1 (N)/Kleiner Physiksaal (N 020)

Start: 15.04.2024 - End: 15.07.2024

### **Tutorials:**

Dr. Martin Lee, E-mail: [martin.lee@lmu.de](mailto:martin.lee@lmu.de)  
Tutorial 1: Fri. 8:30am - 10:00am, Geschw.-Scholl-Pl. 1 (N) / Kleiner Physiksaal (N 020)  
Tutorial 2: Fri. 10:30am - 12:00am, Geschw.-Scholl-Pl. 1 (N) / Kleiner Physiksaal (N 020)

Start: 26.04.2024 - End: 19.07.2024

# Lecture Structure

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## **Tutorials and grading:**

Your total grade will be composed of your active participation in the lecture and tutorial (30%), a presentation with questions that you'll give about a research topic based on several research papers (50%) and 3 exercise sheets that will be solved by the students in the tutorials (20%).

Credit: 6 ECTS.

## **Recommended text books and study materials:**

Mikhail I. Katsnelson: " The Physics of Graphene" (Cambridge University Press).

Phaedon Avouris, Tony F. Heinz, Tony Low: "2D Materials: Properties and Devices" (Cambridge University Press).

Hideo Aoki, Mildred S. Dresselhaus: "Physics of Graphene" (Springer).

Steven M. Girvin, Kun Yang: "Modern Condensed Matter Physics" (Cambridge University Press).

Thomas Heinzel: "Mesoscopic Electronics in Solid State Nanostructures" (Wiley-VCH).

Lecture materials and notes are provided PDFs/PowerPoint slides that will be updated every week.

# Lecture Structure

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1. Lecture 15.04 – Introduction of the rich phenomenology of graphene and 2D materials
2. Lecture 22.04 – Tight binding band-structure of graphene, bilayer graphene, hBN and TMDs  
(Exercise 26.04) – Discussion and setting of the dates of the presentations.
3. Lecture 29.04 – Dirac equation, relativistic massless electrons, pseudo-spin texture  
(Exercise 03.05) – Seminar session
4. Lecture 06.05 – Nano-fabrication, nano-characterization and cryogenic techniques  
(Exercise 10.05) – Seminar session
5. Lecture 13.05 – Electronic transport (Drude), 2-terminal vs. 4-terminal measurements, van der Pauw technique,  $I/V$  and  $dI/dV$ , contact resistance, electric field effect, Hall effect and carrier density extraction, effect of disorder, electron mobility, mean free path, substrate effects.  
(Exercise 17.05) – Return and solution of the homework 1
6. Lecture 27.05 – Consequences of the Dirac equation, Klein tunneling, C2T symmetry protected Dirac cones,  $SU(4)$  spin/valley symmetric properties of graphene, symmetry breaking (ZOOM or move to next days).  
(Exercise 31.05) – Seminar session
7. Lecture 03.06 – Relativistic Quantum Hall effect in graphene,  $\pi$ -Berry's phase, Landau Fan diagram, Cyclotron resonance of massless fermions, Zeeman splitting and QH ferromagnetism (ZOOM or move to next days).  
(Exercise 07.06) – Seminar session

# Lecture Structure

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8. Lecture 10.06 – Graphene-like models as starting point for topologically non-trivial phases, Berry's phase, Haldane model, topological invariants, topological Insulators  
(Exercise 14.06) – Return and solution of the homework 2
  
9. Lecture 17.06 – Displacement field driven strong electronic interactions in AB and ABC graphene, introduction to flat band physics, symmetry breaking, half metallicity, superconductivity  
(Exercise 21.06) – Seminar session
  
10. Lecture 24.06 – Band engineering with 1D and 2D super potentials, moiré patterns in graphene on hBN, Hofstadter butterfly, twisted bilayer graphene, Bistrizer-MacDonald model, twisted and lattice-mismatched moiré TMD bilayers  
(Exercise 28.06) – Seminar session
  
11. Lecture 01.07 – Magic angle twisted bilayer graphene – correlated insulators, magnetism, cascade of phase transitions  
(Exercise 05.07) – Return and solution of the homework 3
  
12. Lecture 08.07 – Magic angle twisted bilayer graphene – topology, quantum geometry, unconventional superconductivity, strange metallicity  
(Exercise 12.07) – Seminar session
  
13. Lecture 15.07 – Other emergent moiré systems, t-MoTe<sub>2</sub>/MoTe<sub>2</sub>, ABC graphene/hBN, fractional Chern insulators  
(Exercise 19.07) – Seminar session

# Lecture Structure

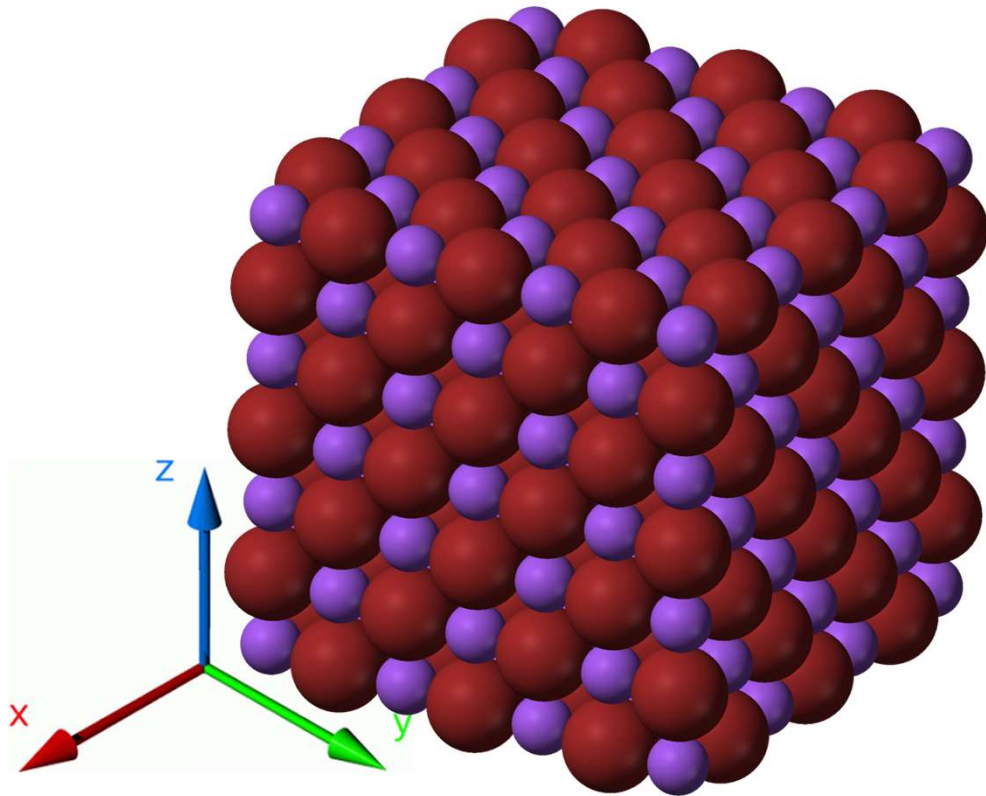
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## Exercises:

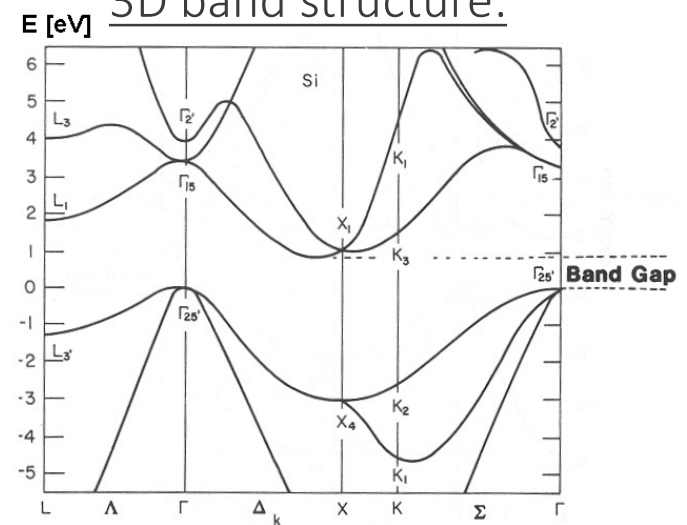
1. 29.04 - 17.05 – Tight binding, Dirac equation, massless electrons, valley degeneracy and pseudo-spin texture, DOS calculation, conversion of carrier concentration to Fermi energy etc.
2. 27.05 – 14.06 – Electronic transport, consequences of the Dirac equation, Klein tunneling, conversion of FWHM into disorder broadening, derivation of LL dispersion vs. carrier concentration and energy etc.
3. 17.06 – 05.07 – Quantum Hall effect, Topological phases, Haldane model, superlattices etc.

# Solid state physics – 3D crystals

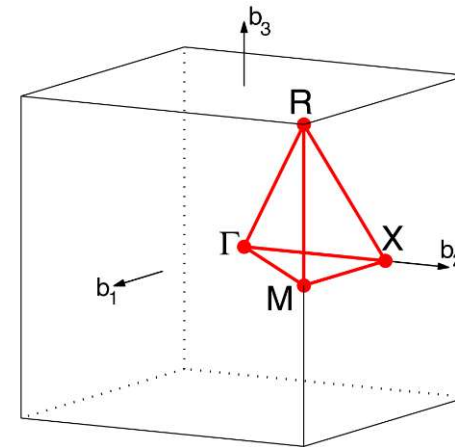
3D crystal lattice:



3D band structure:



3D Brillouin zone:



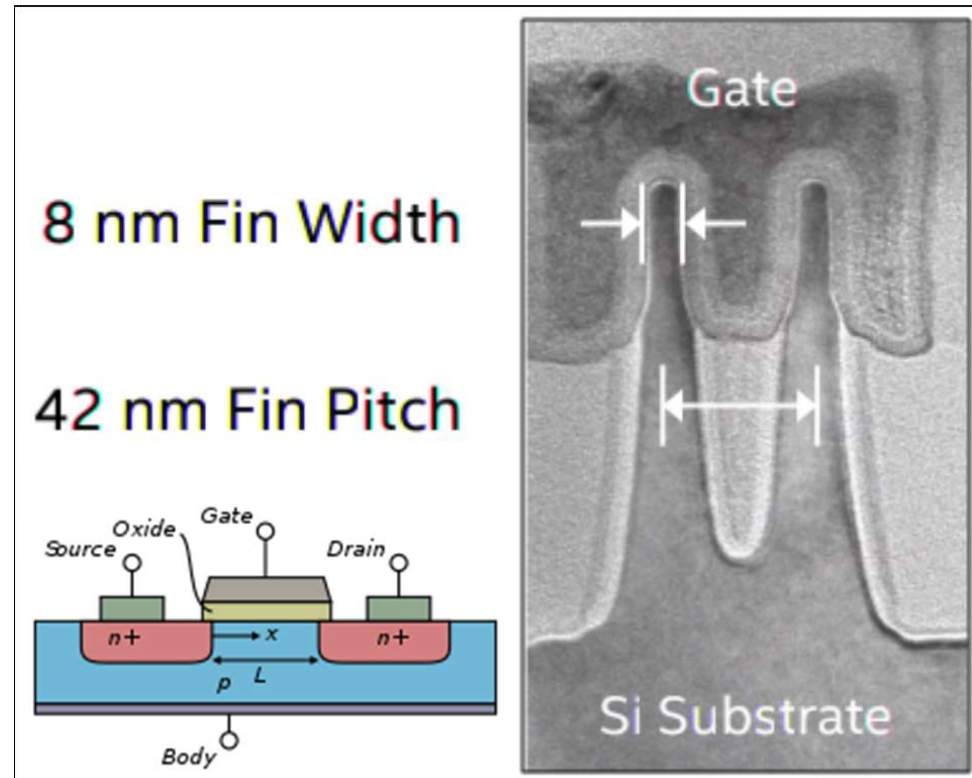
→ Quantum Physics defines crystal structure and electronic properties of a material (metal, insulator, semiconductor).



# Modern Electronics –semiconducting silicon '50

First field effect transistor:

IBM field effect transistor 2020:



→ Research and development led to miniaturization over 7 orders of magnitude from cm to nm scale.

# Modern Electronics –semiconducting silicon '20

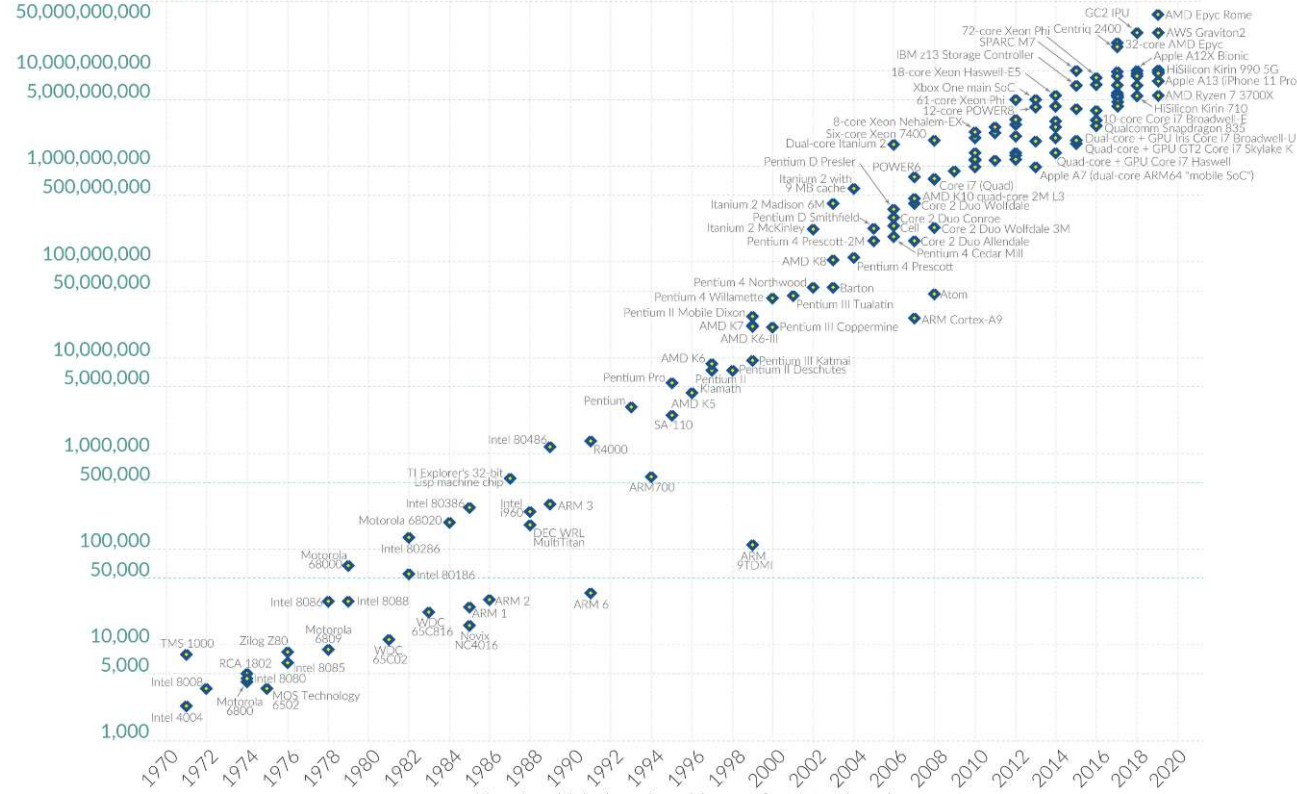
## Moore's Law:

### Moore's Law: The number of transistors on microchips doubles every two years

Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

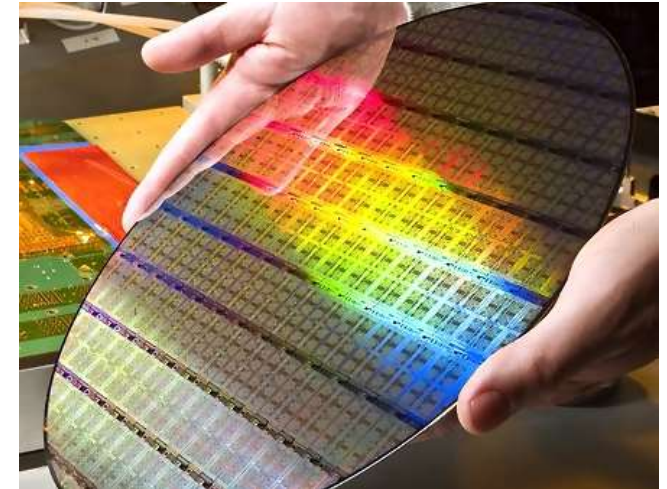


#### Transistor count



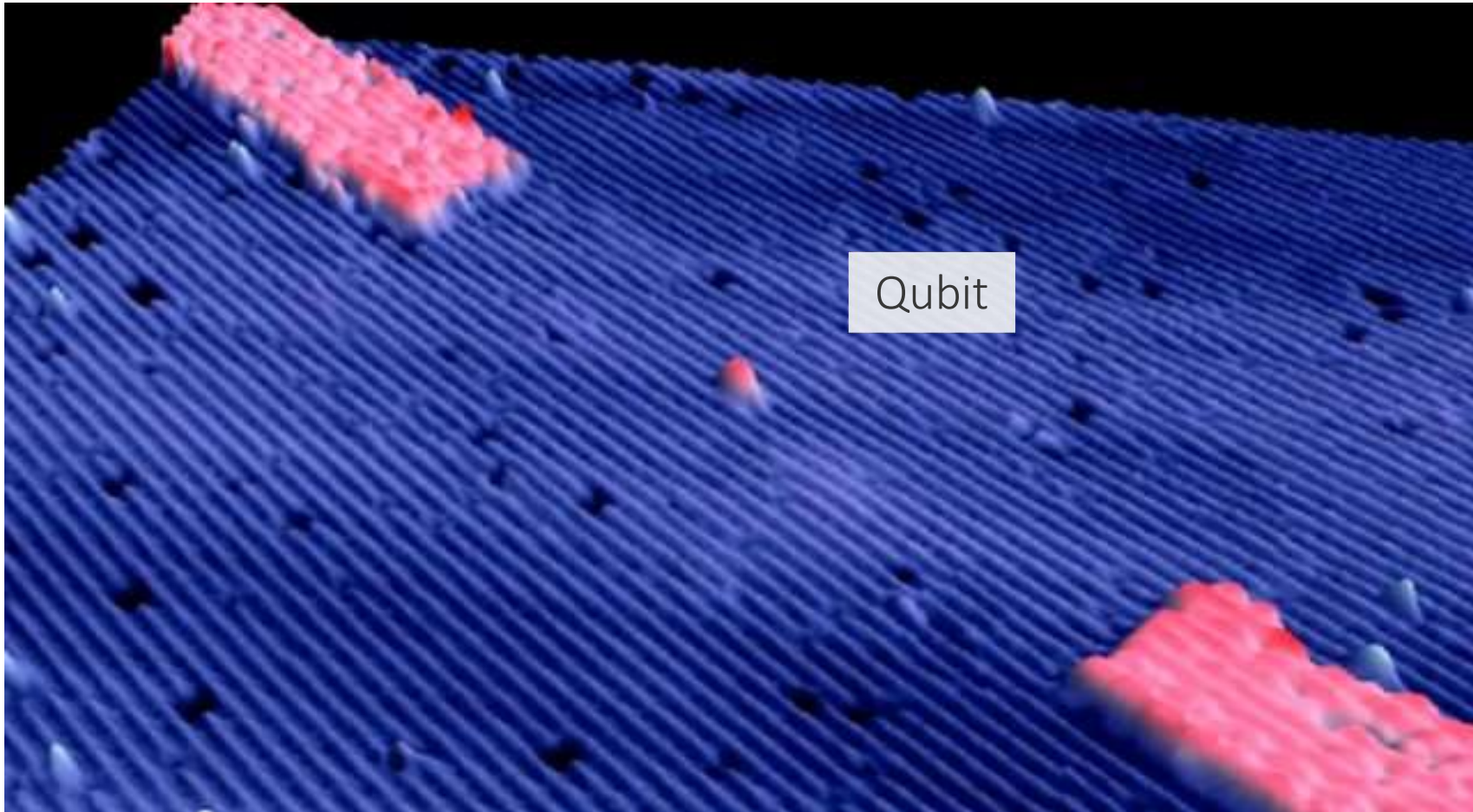
Data source: Wikipedia (wikipedia.org/wiki/Transistor\_count) Year in which the microchip was first introduced  
 OurWorldinData.org – Research and data to make progress against the world's largest problems. Licensed under CC-BY by the authors Hannah Ritchie and Max Roser.

## Billions of transistors on chip:



# Quantum Technologies

Image of a single electron silicon transistor:



- Transistors made of a single atom
- Quantum mechanics plays key role
- Qubits, can store quantum information

# Heterostructures – combining materials '80

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Semiconductor Heterostructure:

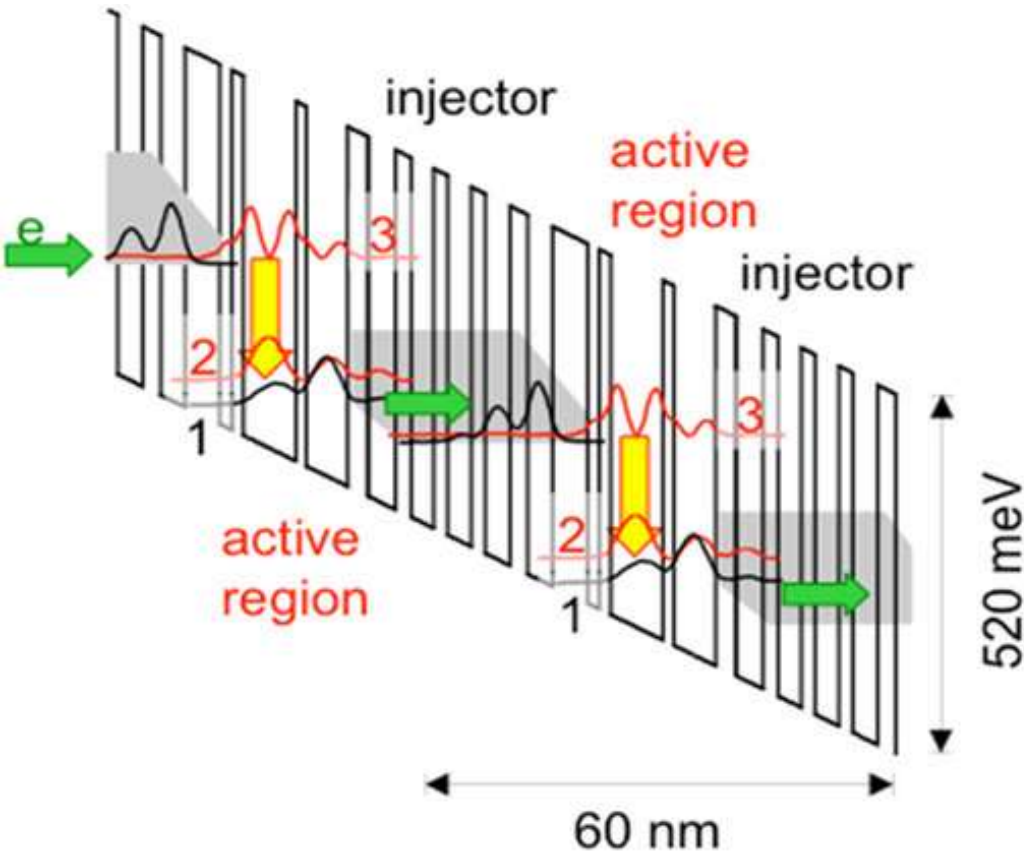
GaAs/AlAs

“The interface is the device.”

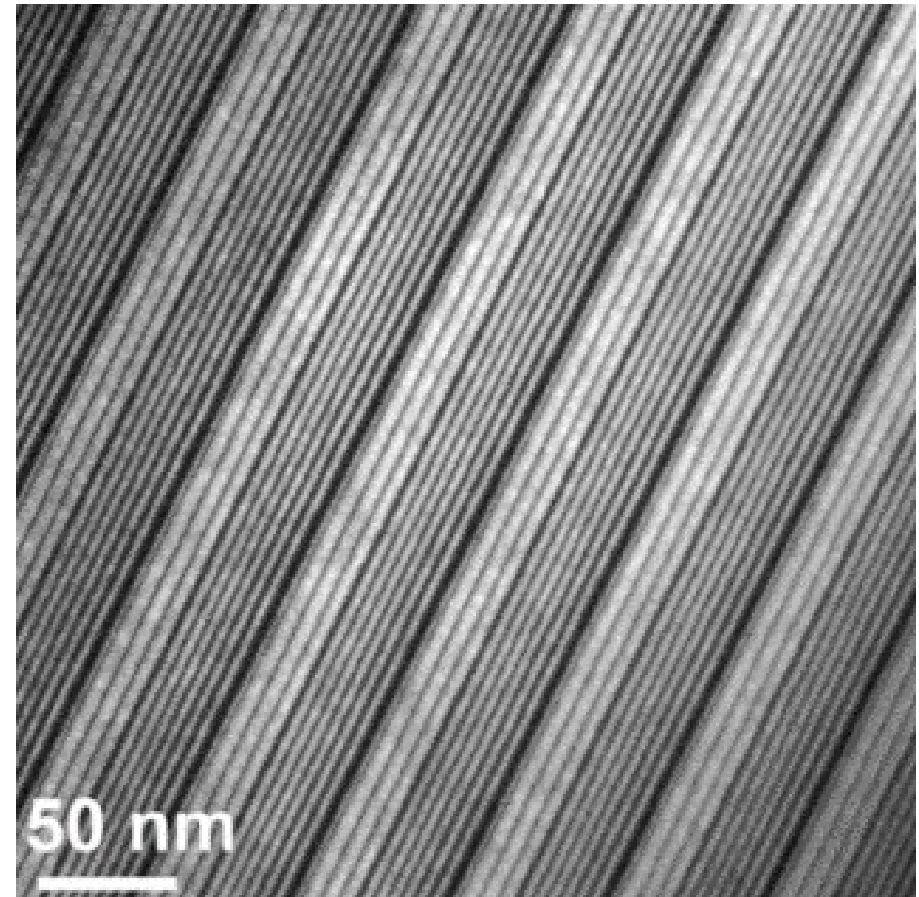
- Combining materials with different properties
  - Interfaces rule

# Quantum Cascade Lasers

Working principle of a QCL:



Cross-section of a QCL:

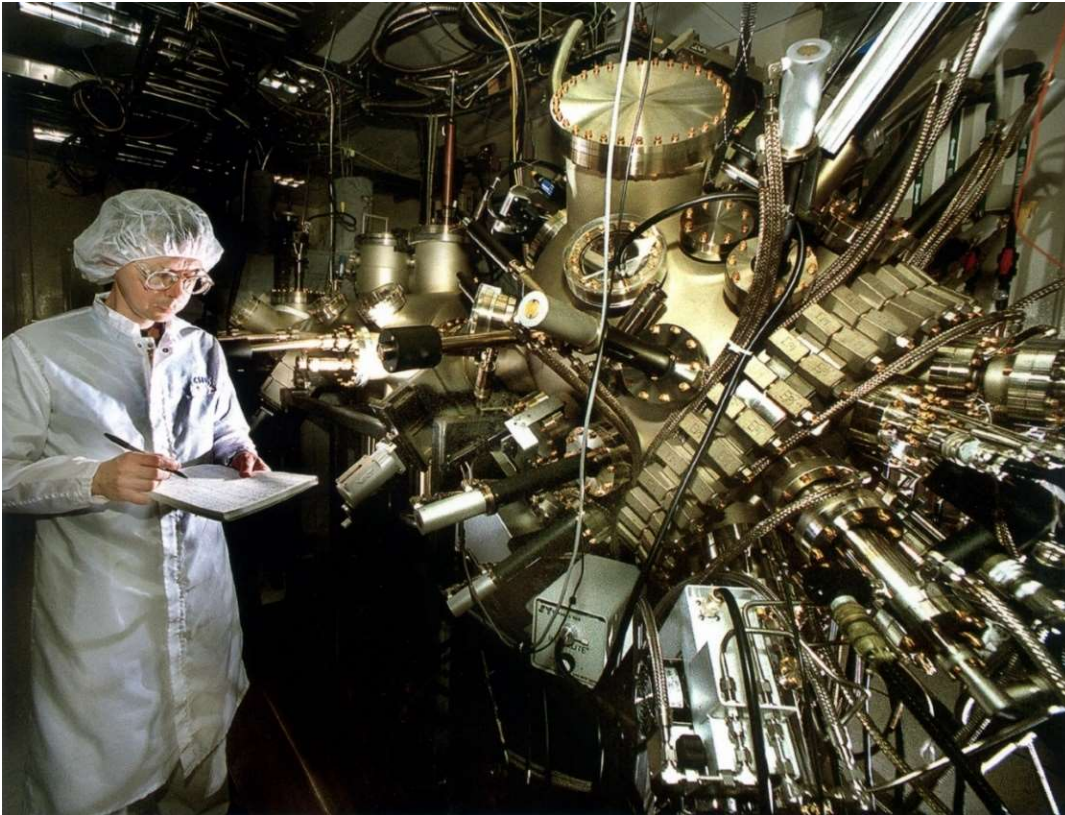


→ Nano-engineered potential wells

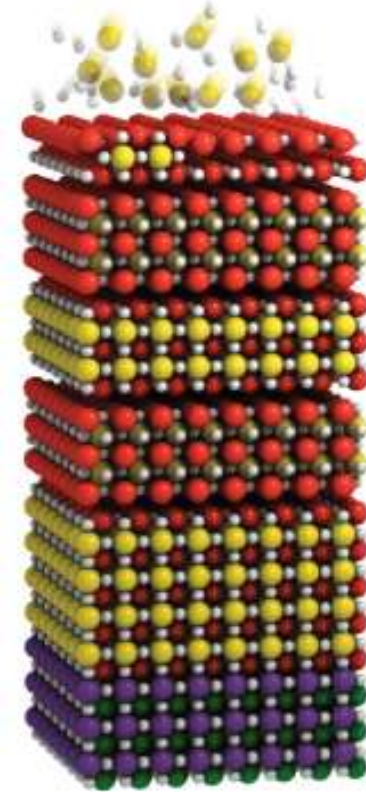
→ Voltage across these, forces electrons to jump to lower energy well

# Molecular beam epitaxy - Molecular LEGO

MBE machine:

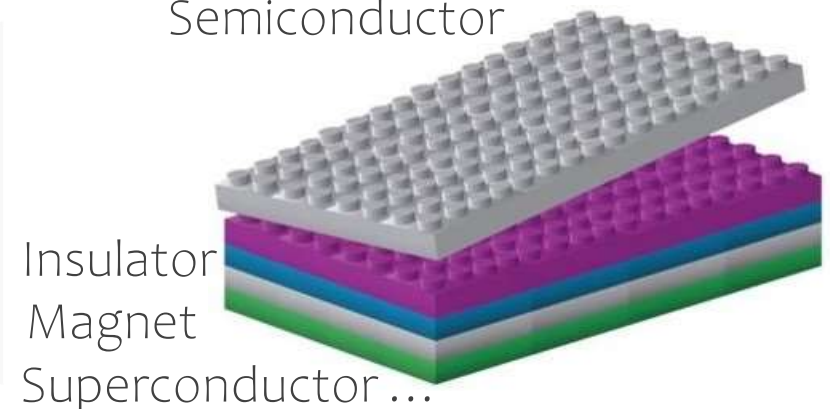


Atom by atom assembly:



Semiconductor

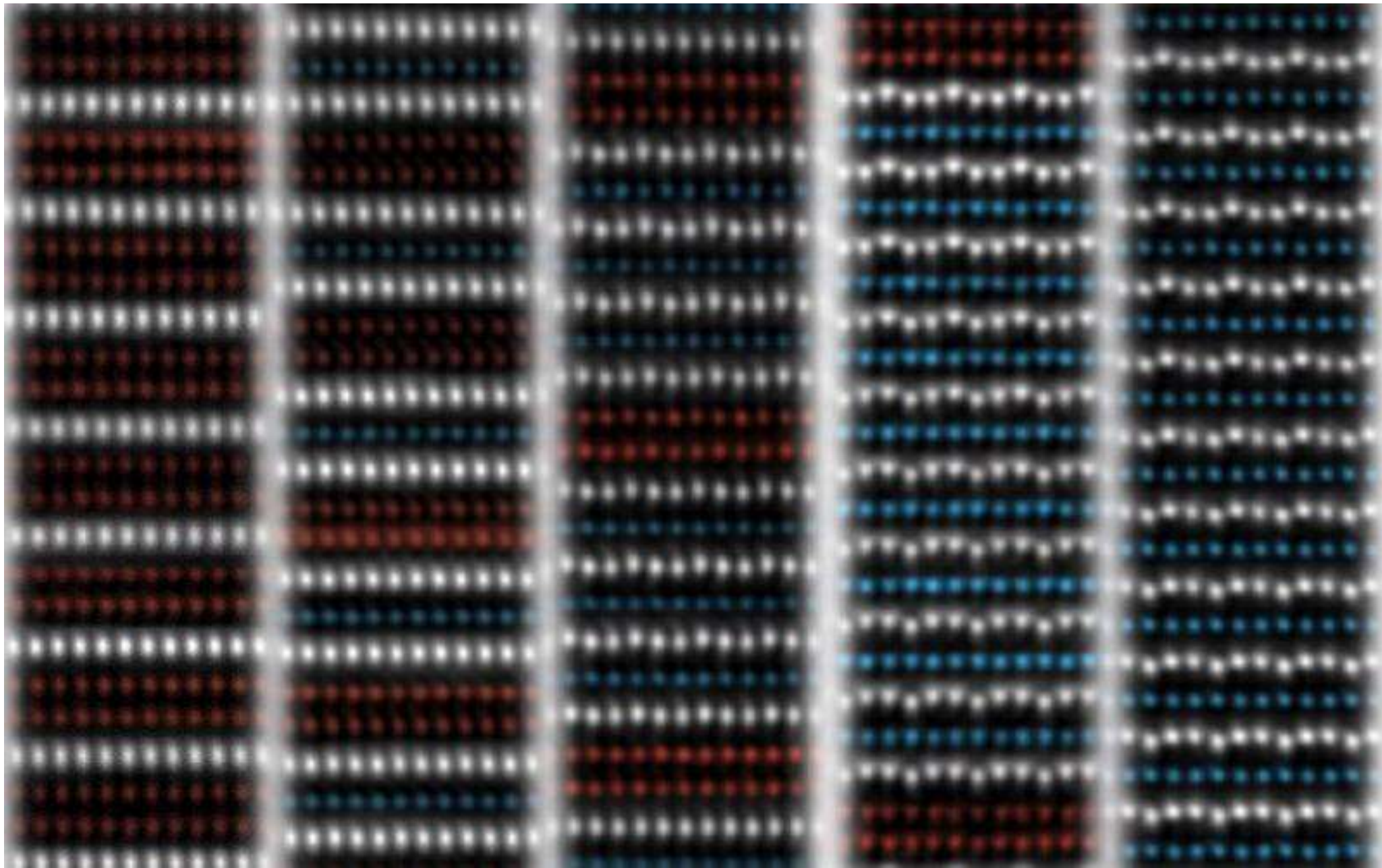
- Atomic engineering.
- Endless combination of properties
- Designer materials



Insulator  
Magnet  
Superconductor ...

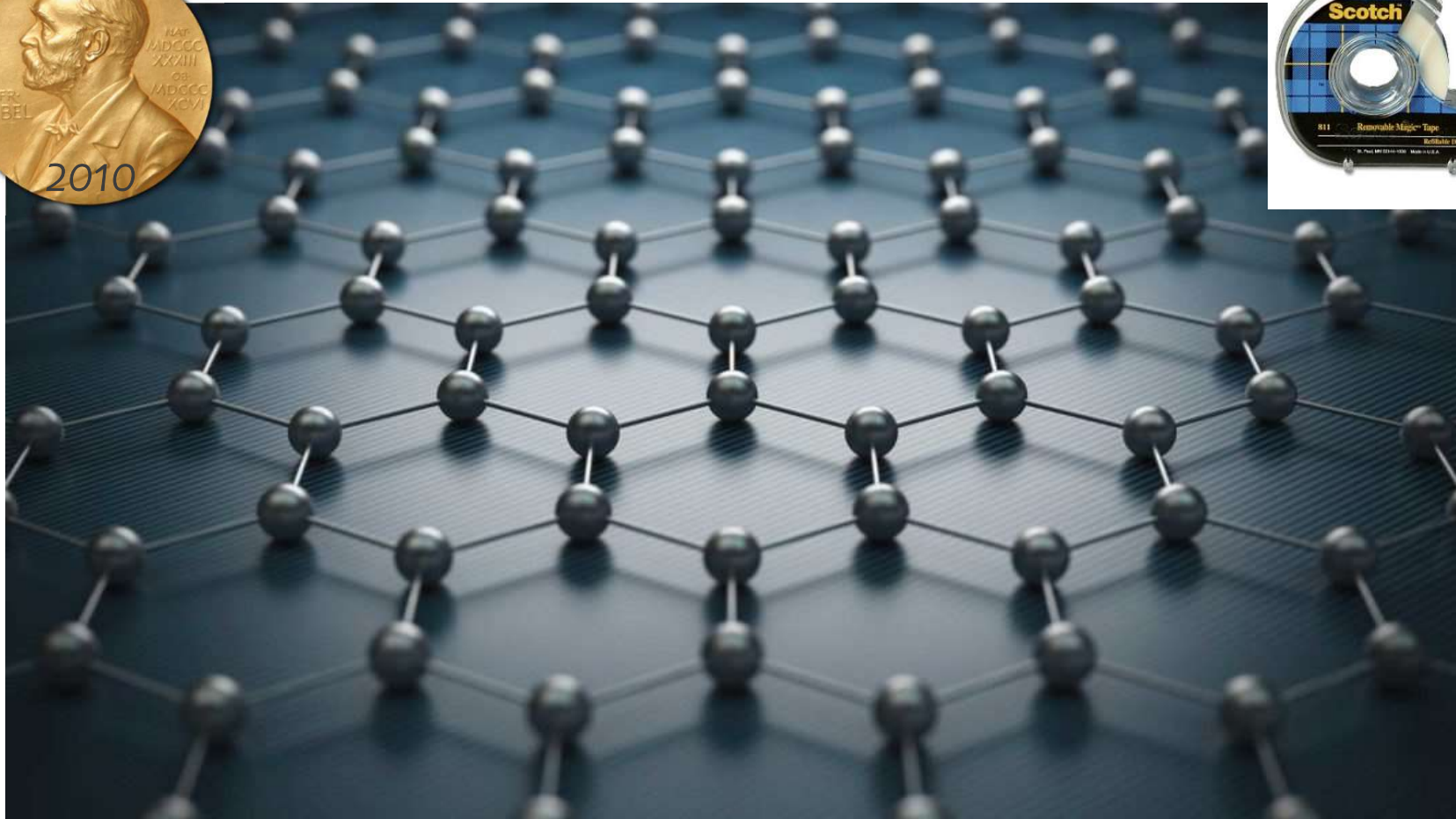
# Heterostructures – Molecular LEGO

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Schlom group (2016).

# Graphene '04

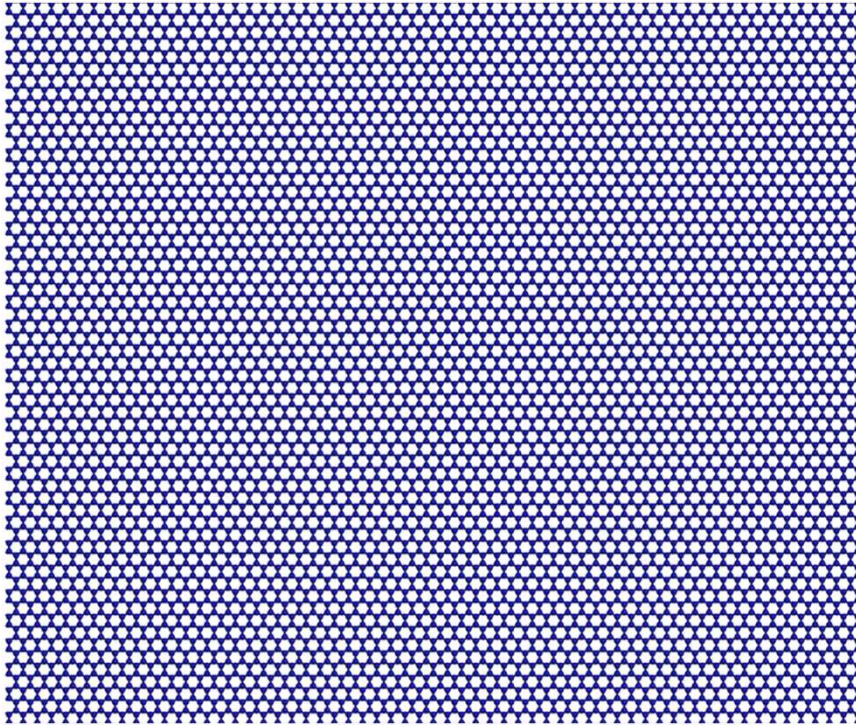


- One atomic layer of carbon atoms
  - True 2D material
  - Scotch tape



# Advent of Graphene and 2D van der Waals materials

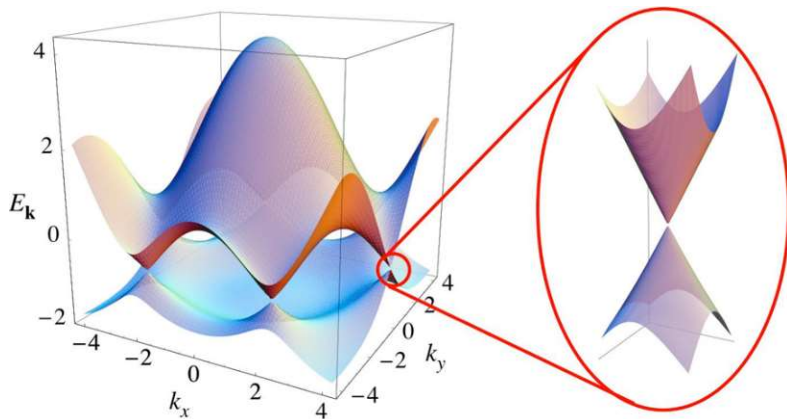
Graphene 2D lattice:



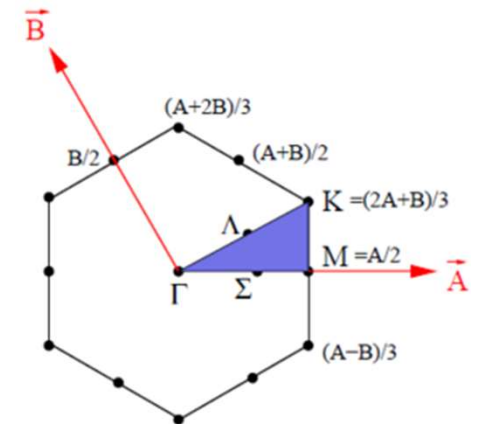
Record properties of graphene :

- Thinnest imaginable material → one atom thick
- Highest surface area → 2630 m<sup>2</sup>/g
- Transparent to light → 97.7 %
- Ultra-broadband absorption → uV - THz
- Stiffest material → 1 TPa
- Strongest material → 130GPa
- Most stretchable material → 20%
- Record thermal conductivity → 6000 W/mK
- Highest current density at RT → 10<sup>6</sup> > copper
- Highest intrinsic mobility → 100 > Si
- Lightest charge carrier → massless Dirac fermions
- Longest mean free path at room temp → microns
- Easily functionalized and process able
- Impervious to even He

Graphene 2D band structure:



Graphene 2D Brillouin zone:



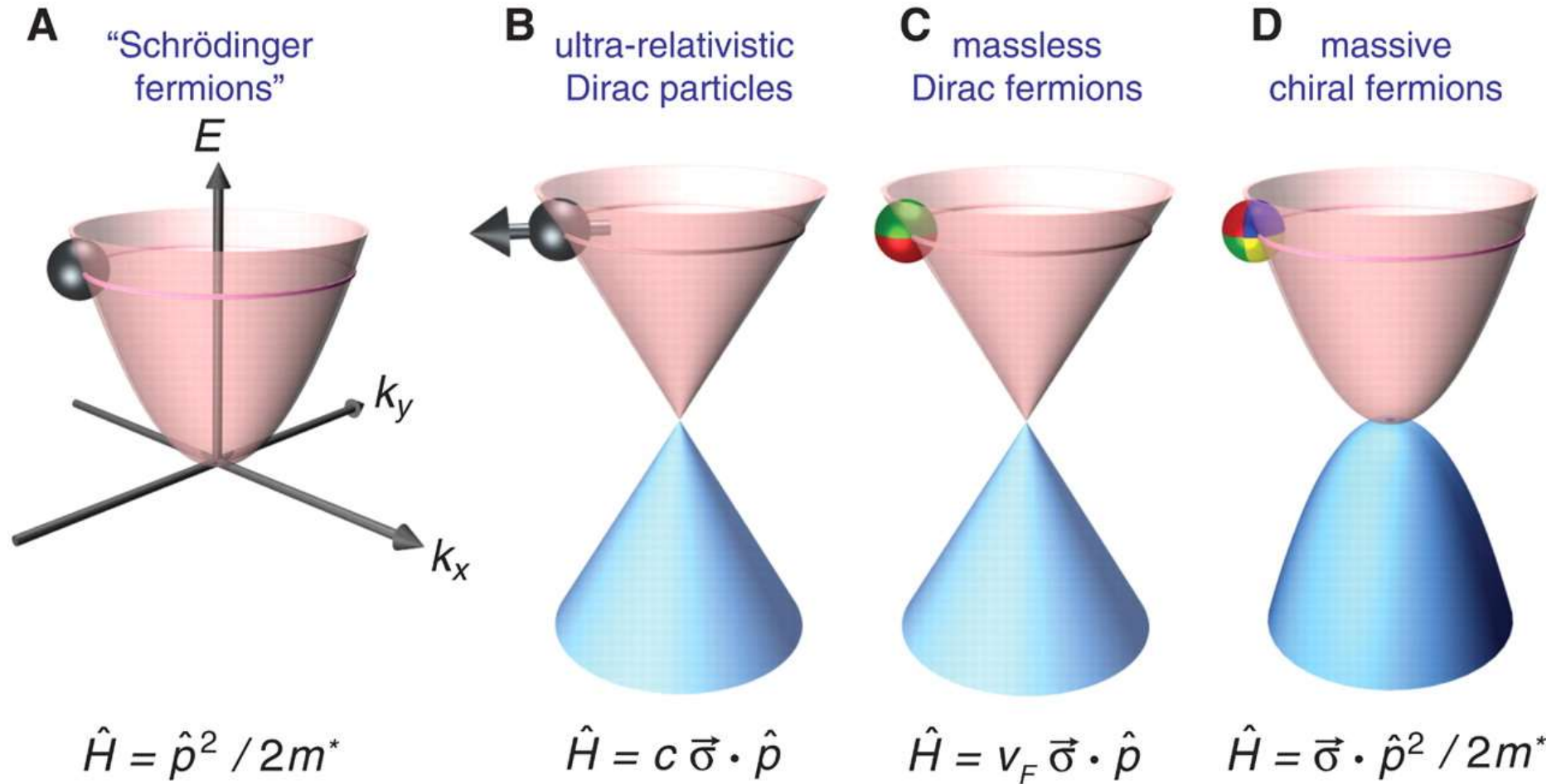
Dirac equation

$$\hat{H} = v_F \vec{\sigma} \cdot \hat{p}$$

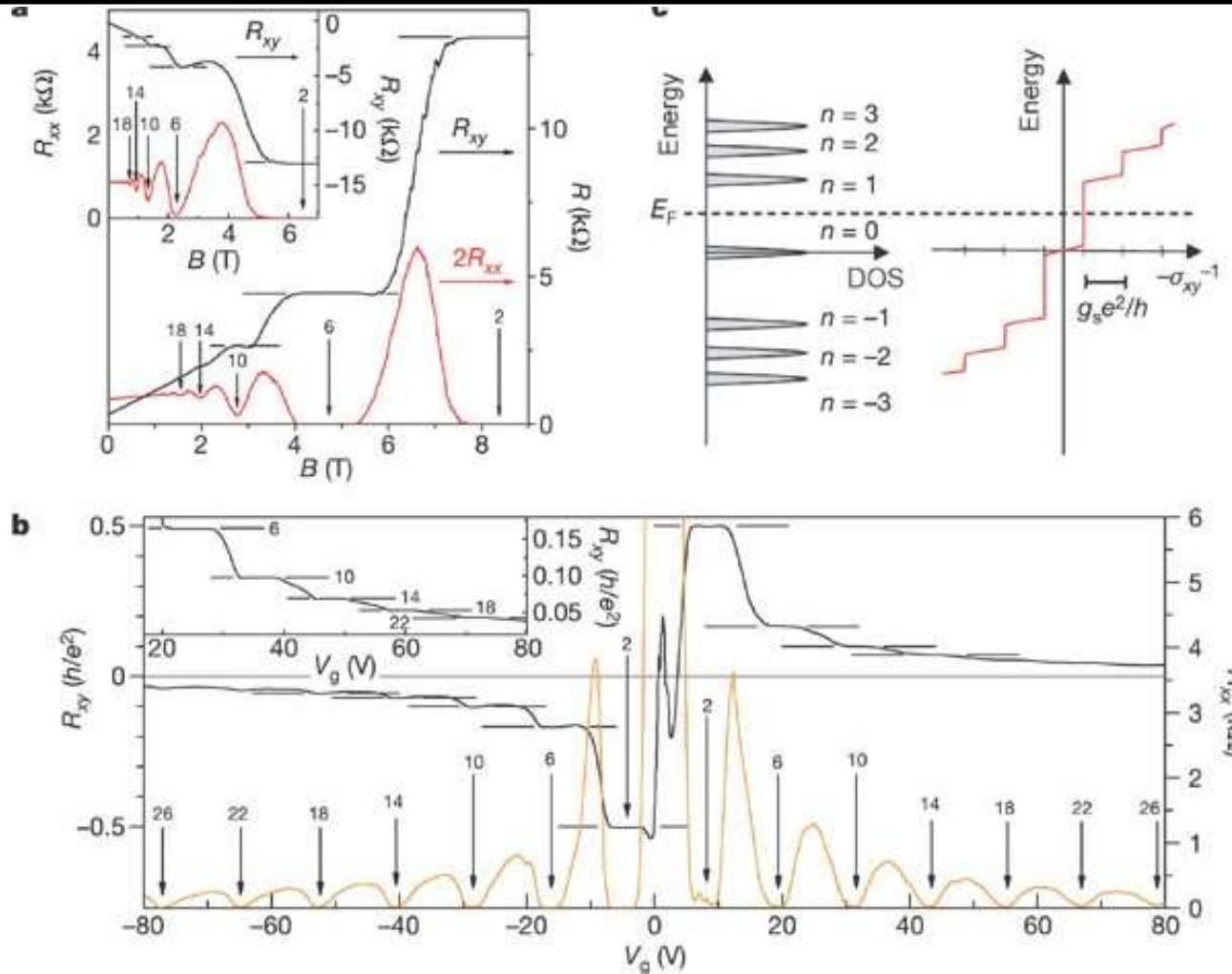
Effective mass

$$m^* = \pm \hbar \left( \frac{d^2 E_k}{dk^2} \right)^{-1}$$

# Exotic new band structures



# Relativistic Integer Quantum Hall effect in graphene

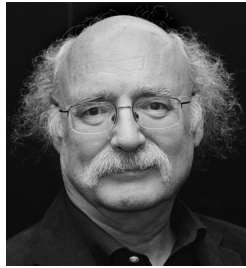


- Clearly an integer QHE with  $R_{xy} = (h/e^2)/n$ , where  $n$  is an integer.
- However, the sequence of LLs is quite different, where  $R_{xy} = (h/e^2)/n$  takes values  $n = 2, 6, 10$  etc.
- This implies a degeneracy of 4 (spin+valley), and a zero-energy LL, which is not present in normal 2DEGs.

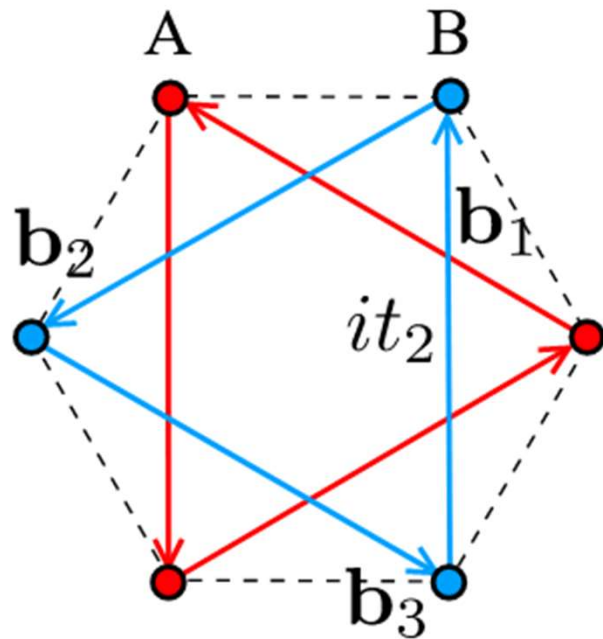
# Haldane model - Topology



2016

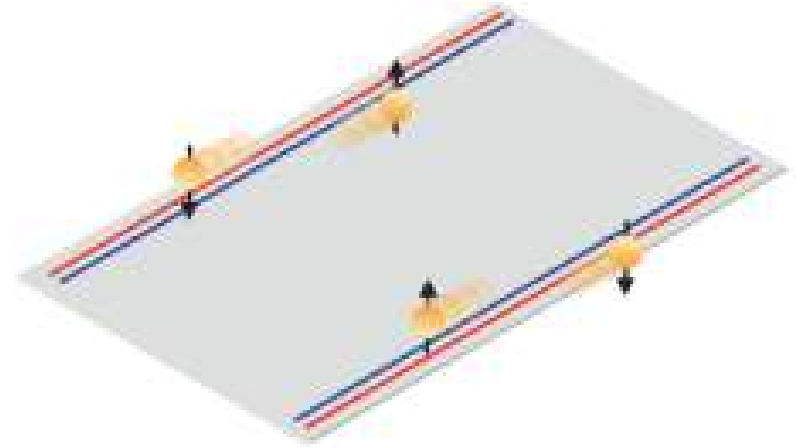


Duncan Haldane

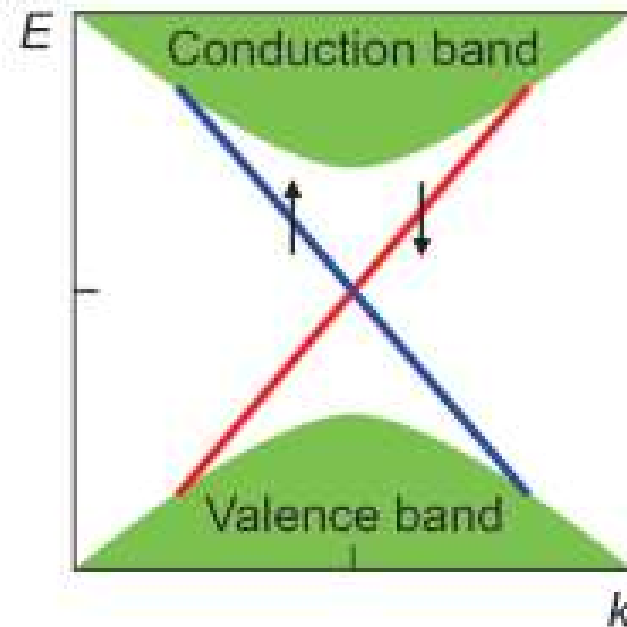


- Next-nearest neighbor hopping induces gap opening
- Berry curvature loops
- Topological Chern bands

(b) 2D topological insulator



(e)



# Quantum Hall Effect without B-field

Hall  
(1879)

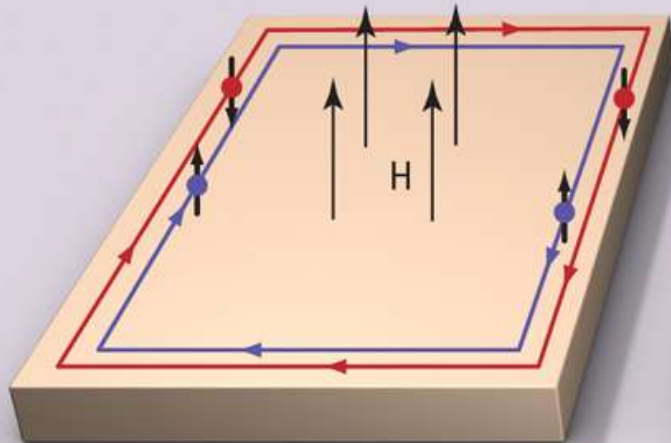
Spin Hall  
(2004)

Anomalous Hall  
(1881)

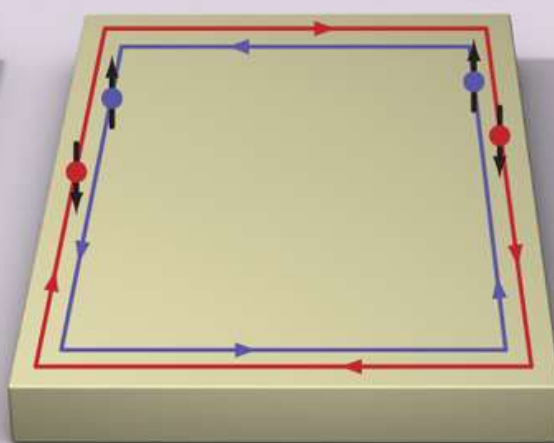
Quantum Hall  
(1980)

Quantum spin Hall  
(2007)

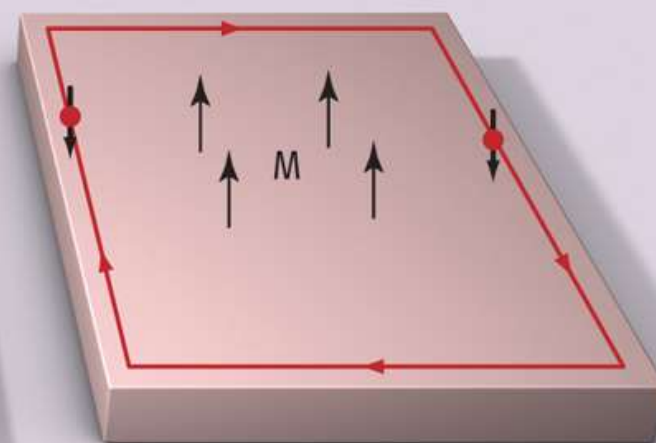
Quantum anomalous Hall  
(2013)



Quantum Hall



Quantum spin Hall



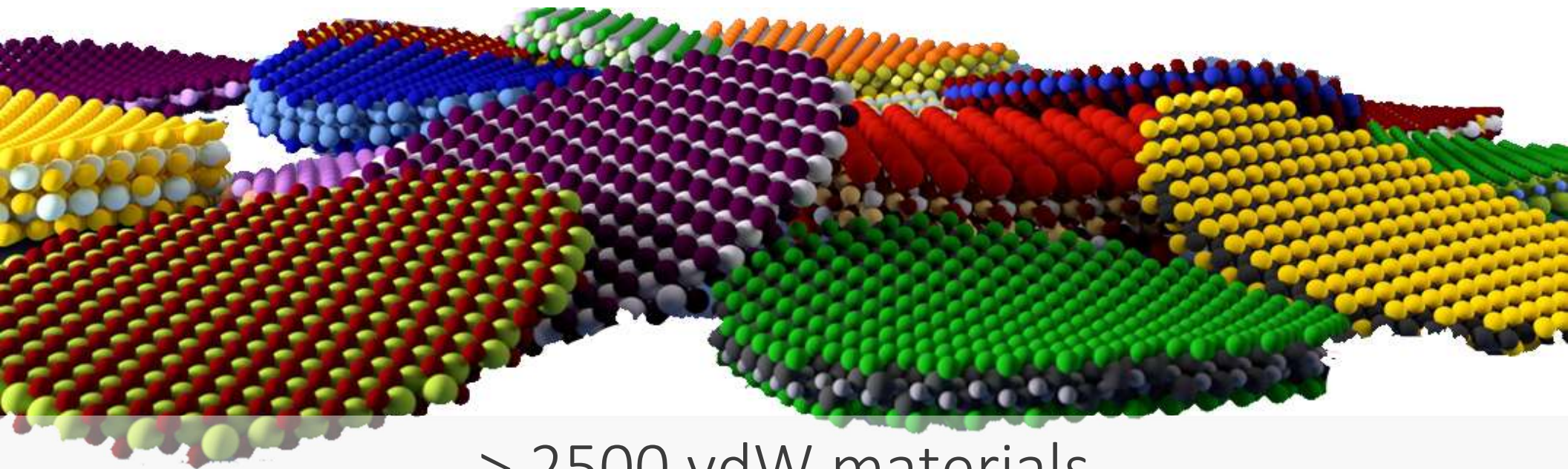
Quantum anomalous Hall

- Quantum Spin Hall effect and Quantum Anomalous Hall effect – Quantum Hall effect in zero magnetic field, given rise by a combination of topological electron states and a strong spin-orbit coupling and/or ferromagnetic bulk of the material.

# Graphene and 2D van der Waals materials

Graphene family	Graphene	hBN 'white graphene'	BCN	Fluorographene	Graphene oxide
2D chalcogenides	MoS <sub>2</sub> , WS <sub>2</sub> , MoSe <sub>2</sub> , WSe <sub>2</sub>		Semiconducting dichalcogenides: MoTe <sub>2</sub> , WTe <sub>2</sub> , ZrS <sub>2</sub> , ZrSe <sub>2</sub> and so on	Metallic dichalcogenides: NbSe <sub>2</sub> , NbS <sub>2</sub> , TaS <sub>2</sub> , TiS <sub>2</sub> , NiSe <sub>2</sub> and so on	
				Layered semiconductors: GaSe, GaTe, InSe, Bi <sub>2</sub> Se <sub>3</sub> and so on	
2D oxides	Micas, BSCCO	MoO <sub>3</sub> , WO <sub>3</sub>	Perovskite-type: LaNb <sub>2</sub> O <sub>7</sub> , (Ca,Sr) <sub>2</sub> Nb <sub>3</sub> O <sub>10</sub> , Bi <sub>4</sub> Ti <sub>3</sub> O <sub>12</sub> , Ca <sub>2</sub> Ta <sub>2</sub> TiO <sub>10</sub> and so on		Hydroxides: Ni(OH) <sub>2</sub> , Eu(OH) <sub>2</sub> and so on
	Layered Cu oxides	TiO <sub>2</sub> , MnO <sub>2</sub> , V <sub>2</sub> O <sub>5</sub> , TaO <sub>3</sub> , RuO <sub>2</sub> and so on			Others

A. Geim, et. al. Nature (2013).

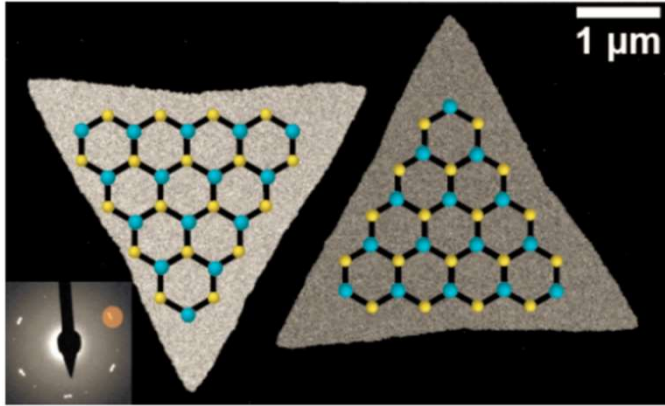


> 2500 vdW materials

# Single layer vdW materials

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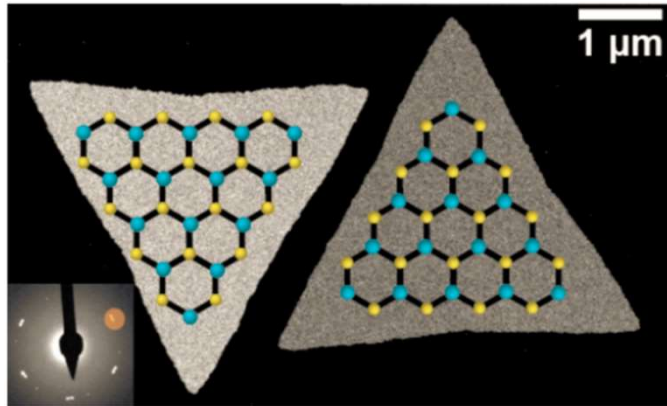
High quality single crystals - 2D physics in  
the clean limit



- True 2D high quality single crystals
- Quantum confinement in the z direction

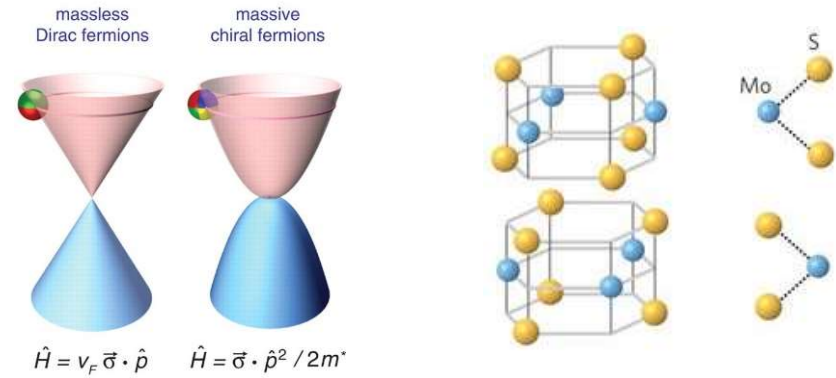
# Single layer vdW materials

## High quality single crystals - 2D physics in the clean limit



- True 2D high quality single crystals
- Quantum confinement in the z direction

## Layer number control – new physical systems

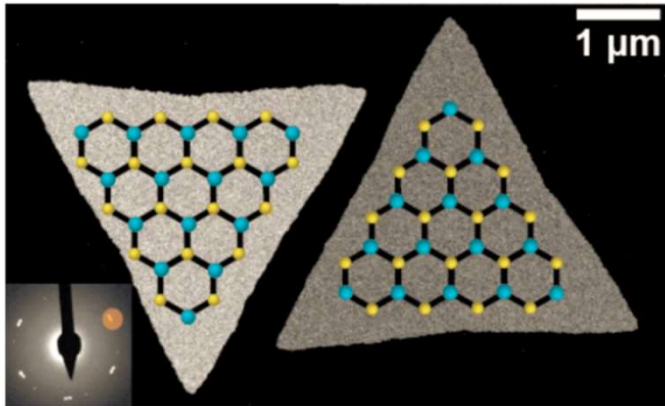


- Altered unit cells → changed degeneracies and broken inversion symmetries
- Altered band-structures below 8 layers
  - Changed phonon spectrum



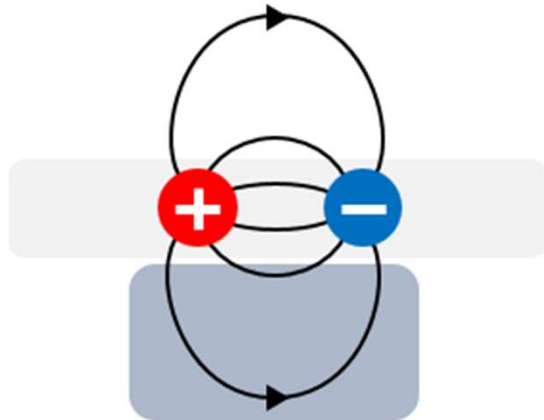
# Single layer vdW materials

## High quality single crystals - 2D physics in the clean limit



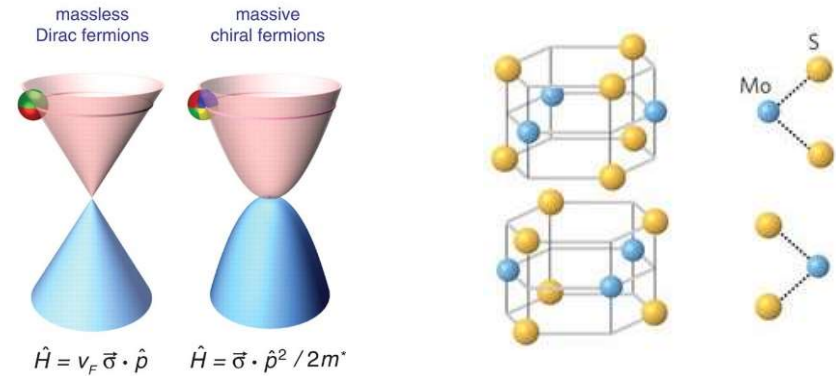
- True 2D high quality single crystals
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## Enhanced electronic interactions



- Reduced environmental screening enhances electronic interactions

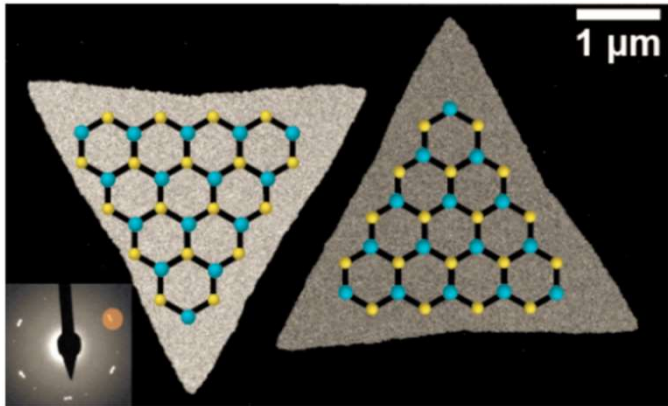
## Layer number control – new physical systems



- Altered unit cells  $\rightarrow$  changed degeneracies and broken inversion symmetries
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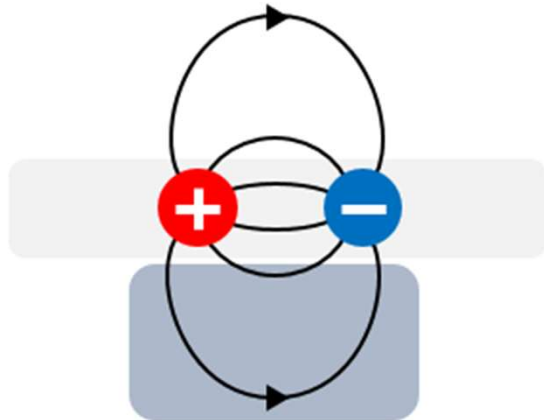
# Single layer vdW materials

## High quality single crystals - 2D physics in the clean limit



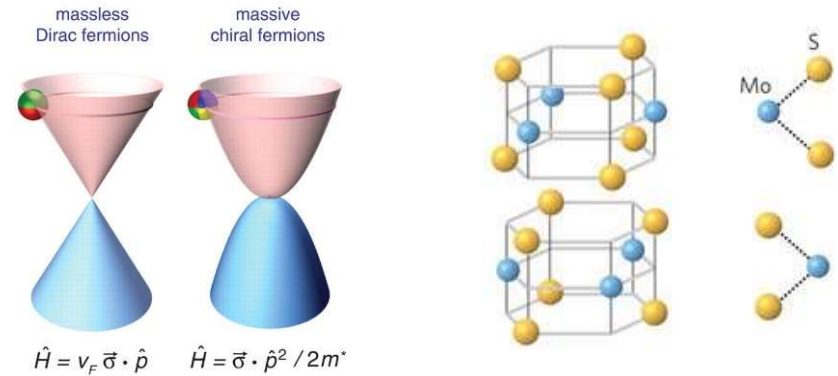
- True 2D high quality single crystals
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## Enhanced electronic interactions



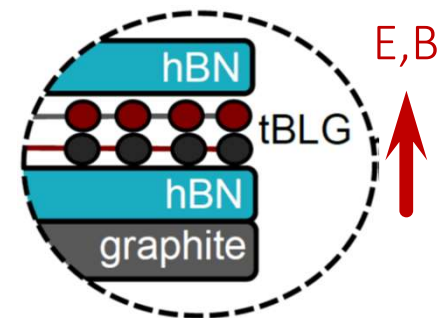
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## Layer number control – new physical systems



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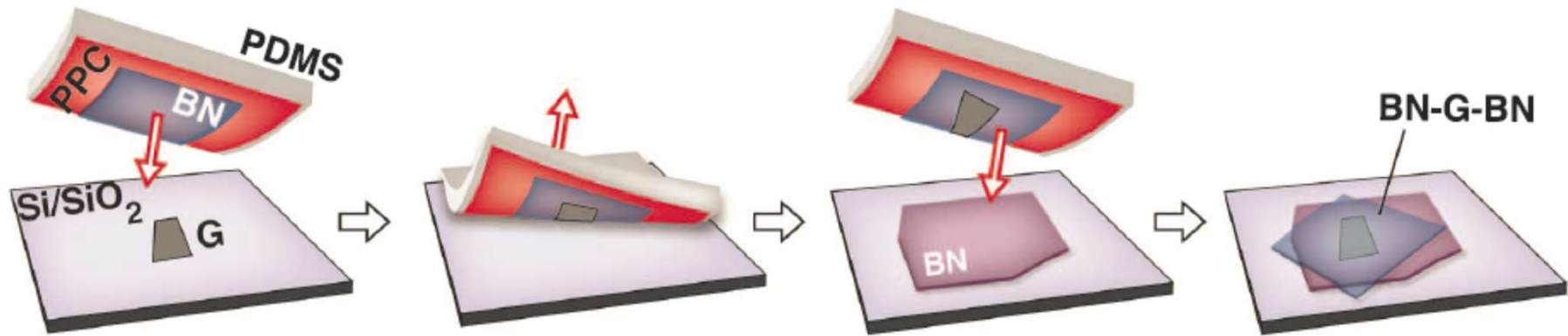
## E- and B-field control



- Electric field control across phase-transitions
- Novel quantum phases in high B-fields

# VdW heterostructures – clean coupling over $< 1\text{nm}$

vdW co-lamination transfer technique:



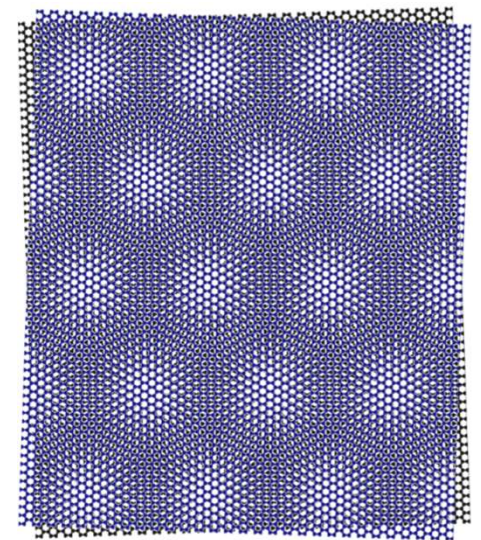
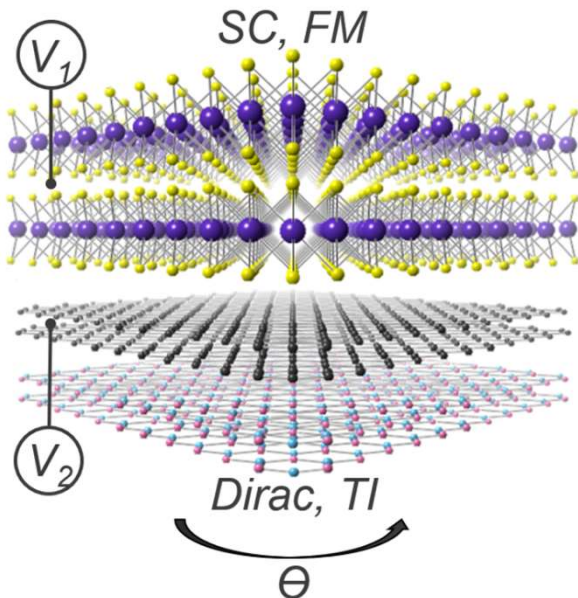
A. Geim, et. al. Nature (2013).

C. Dean, P. Kim, J. Hone, et. al. Science (2013).

Designer vdW stack:

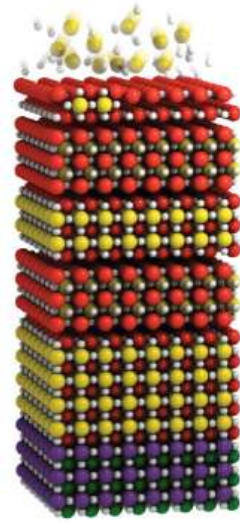
TEM cross-section:

Moiré superlattice:

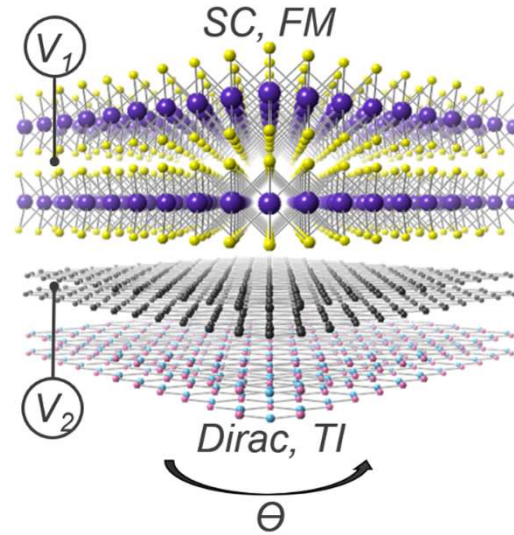


# MBE (LEGO) vs. vdW (CARDS)

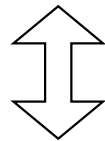
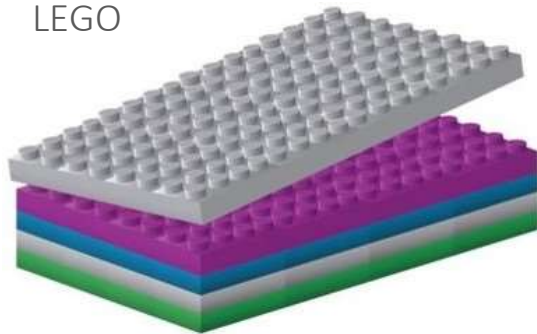
Molecular beam epitaxy:



Van der Waals assembly:



LEGO



Crystallographic orientation locked



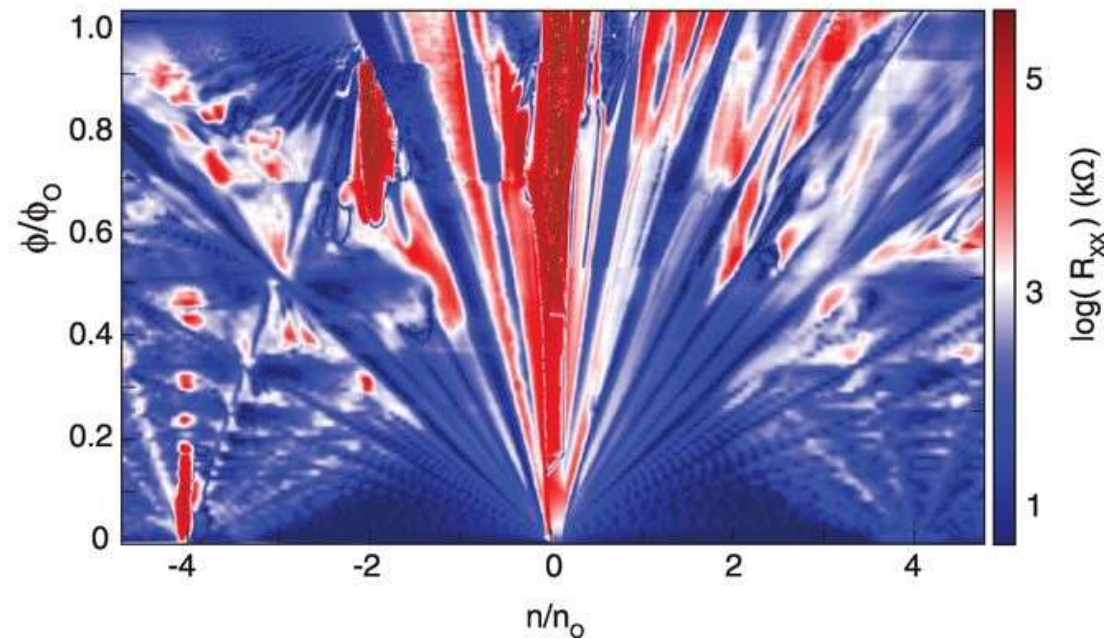
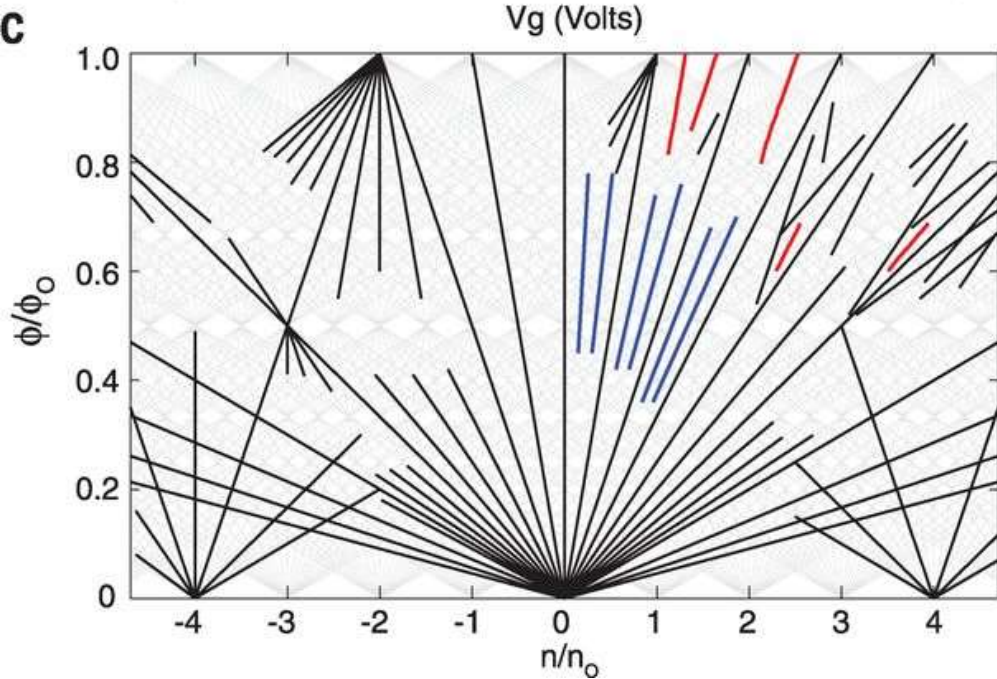
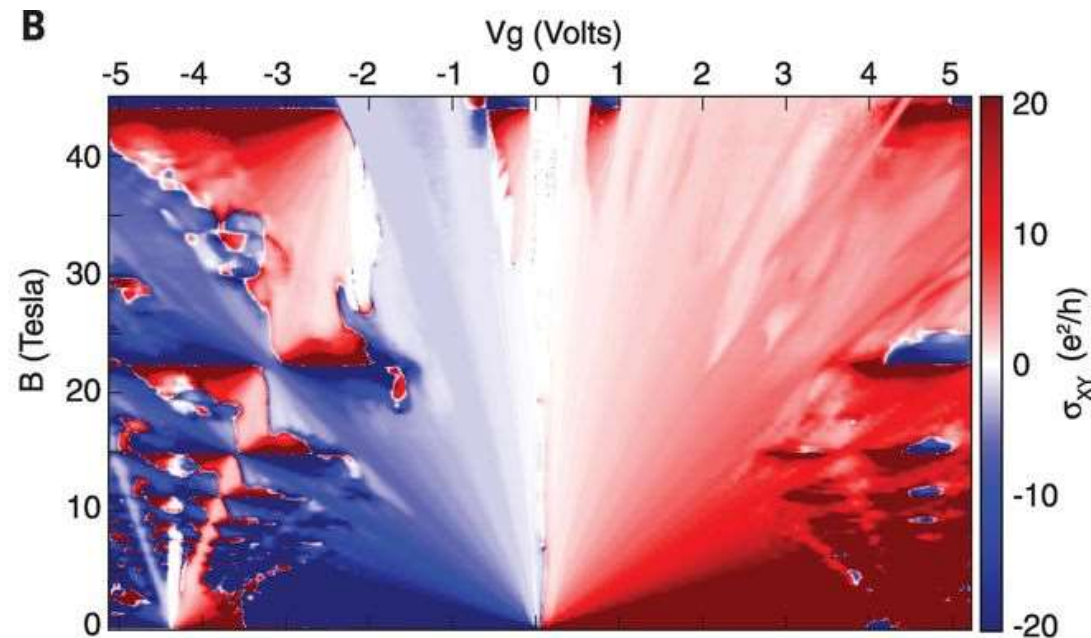
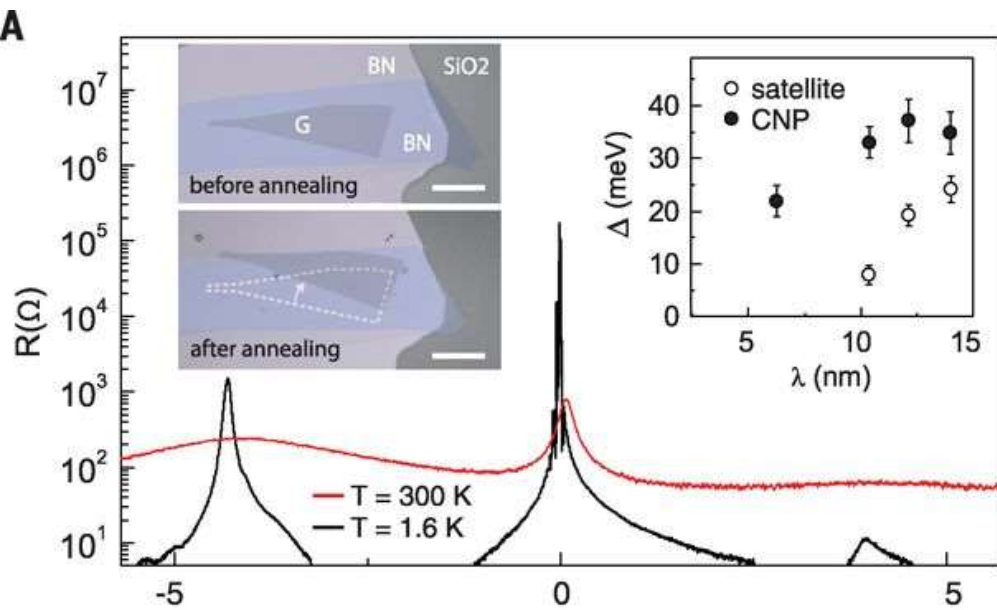
CARDS



Crystallographic orientation free

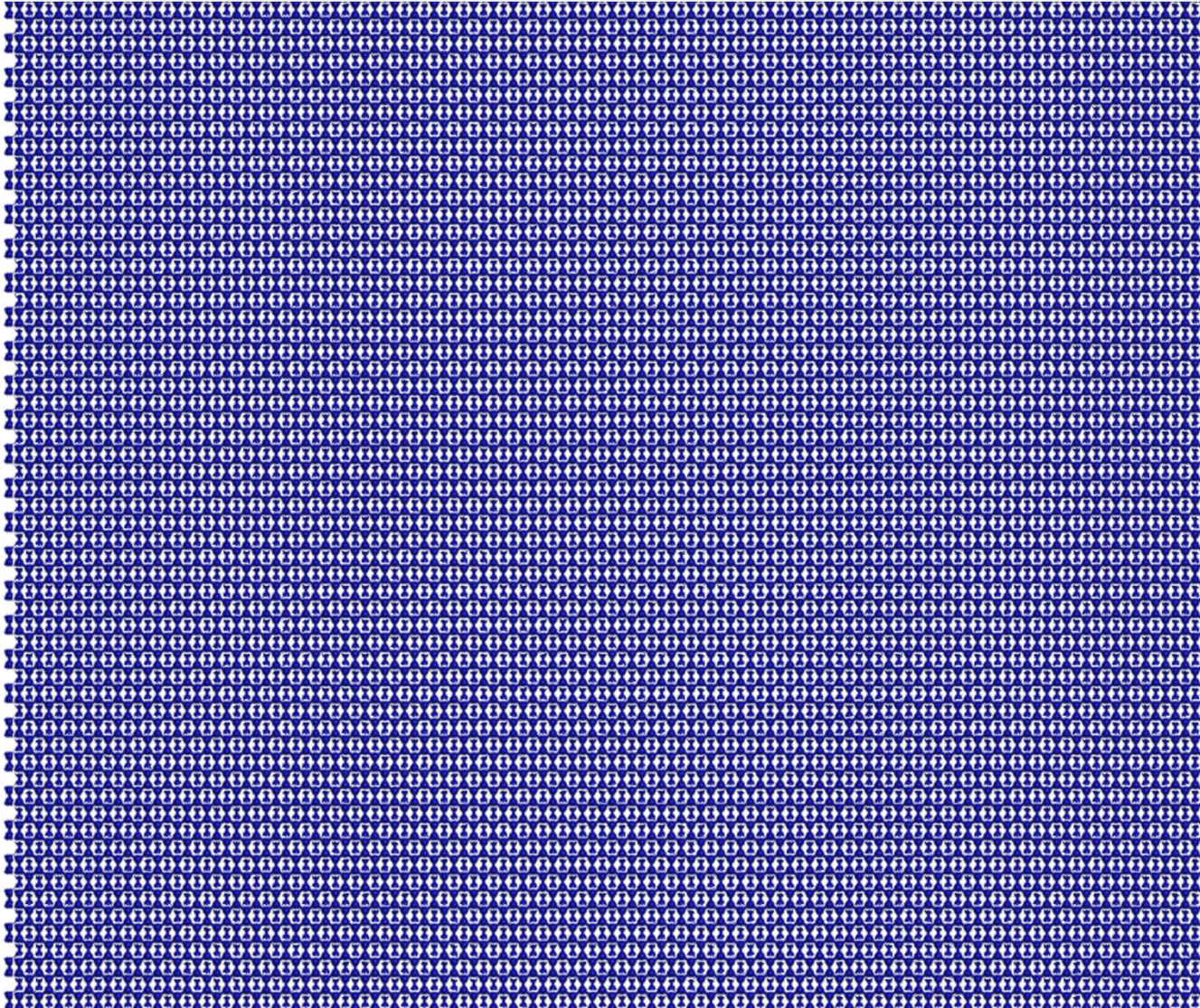
New → Free crystallographic orientation

# Hofstadter butterfly



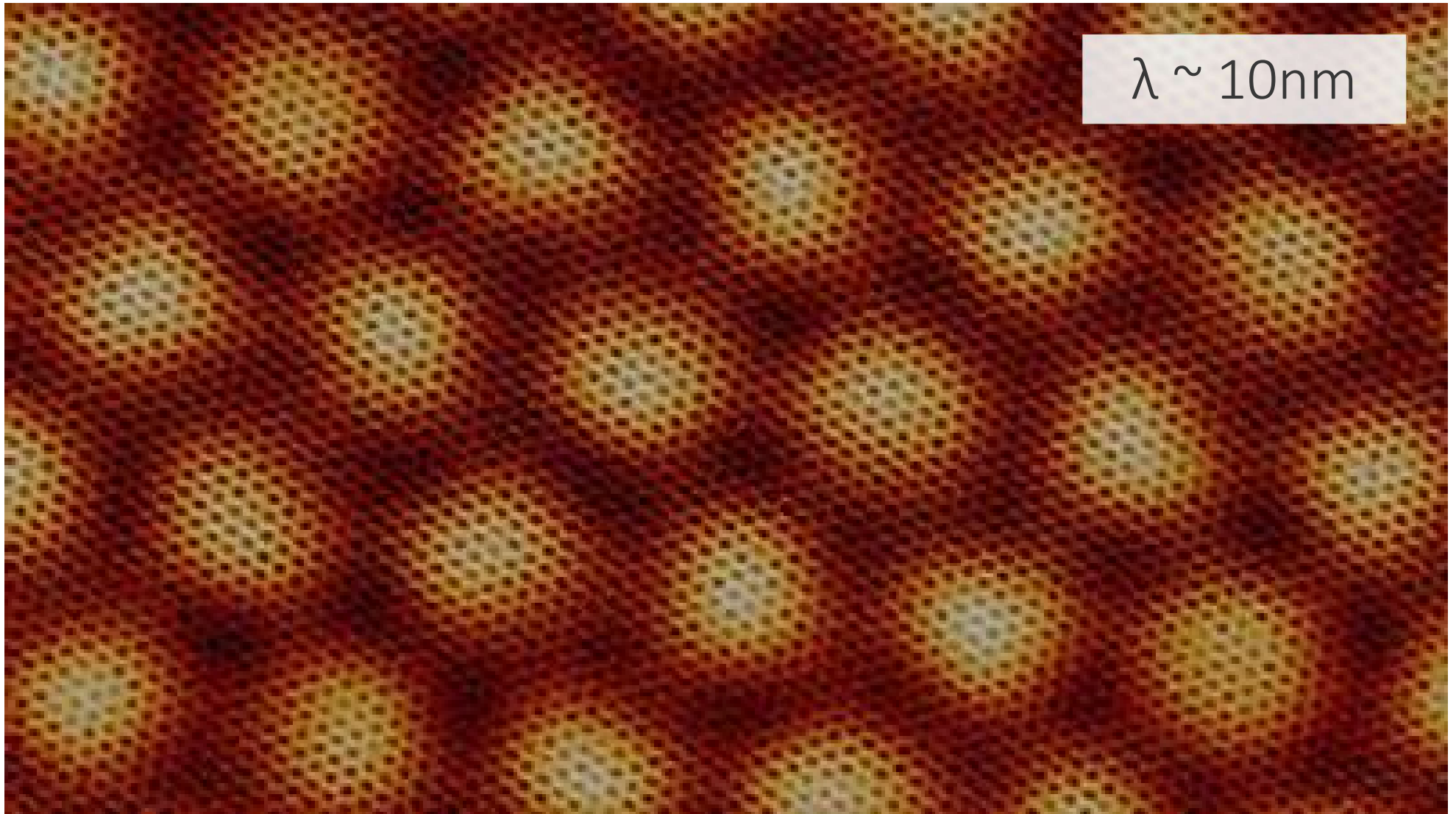
# Twisting 2D materials

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# Moire interference pattern – large scale superpotential

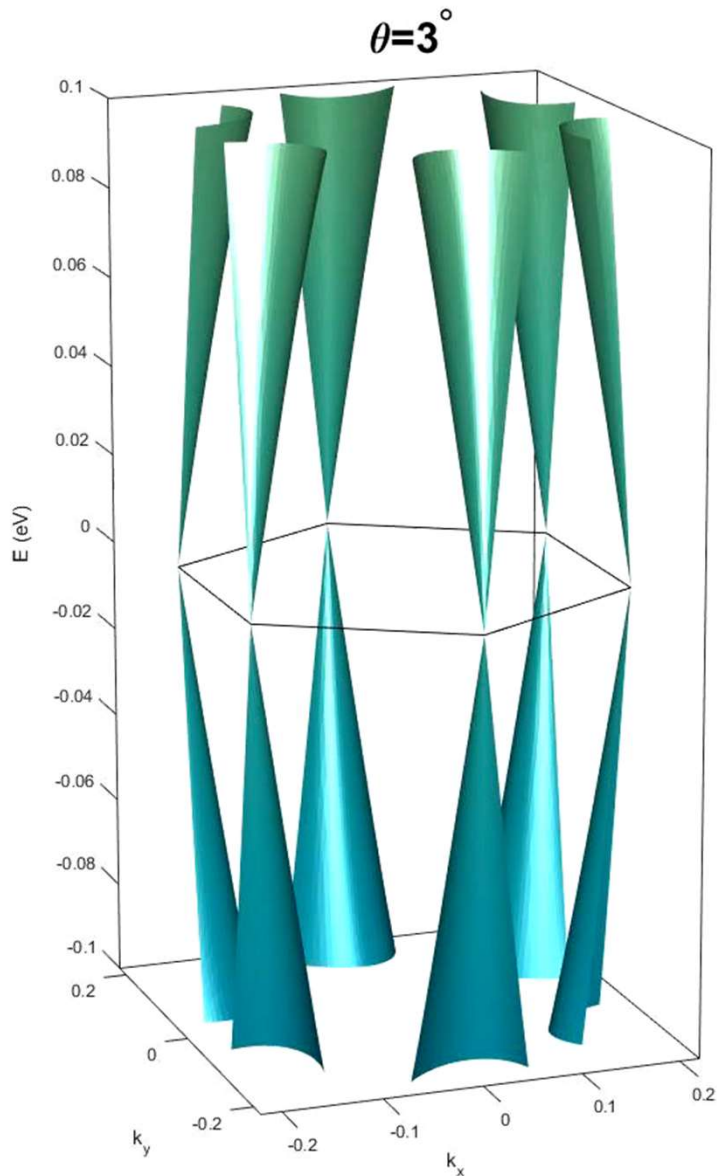
Scanning tunneling microscopy image of twisted bilayer graphene:



Yazdany group (2023).

# Bandstructure evolution and the magic angle of $1.1^\circ$

Jarillo-Herrero group (2018).



Entirely novel quantum phases appear in flat bands at  $1.1^\circ$

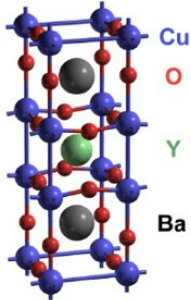
- Mott insulators
- superconductivity
- magnetism
- topology



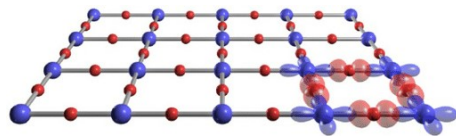
# New platform for strongly correlated physics

## Crystals:

$\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$   
(YBCO)

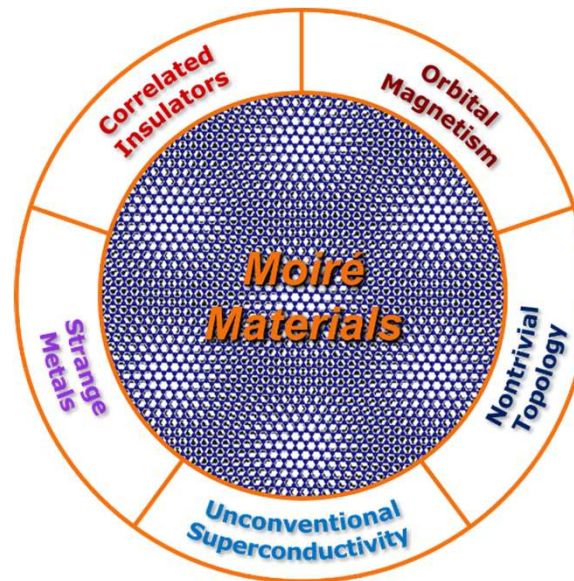


cuprates



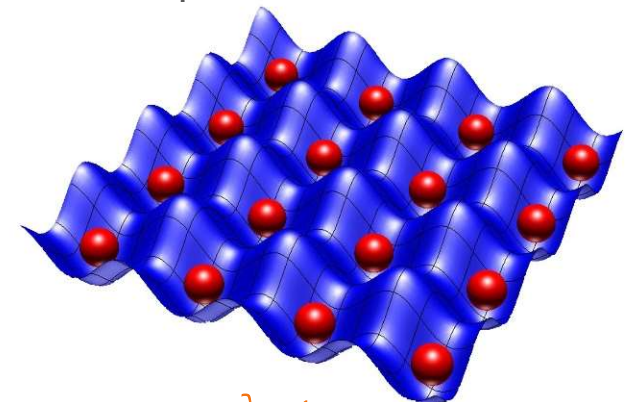
$\lambda \sim 1\text{\AA}$   
 $U \sim \text{eV}$

## Magic-angle tBLG:



$\lambda \sim 10\text{nm}$   
 $U \sim \text{meV}$

## Optical lattices:

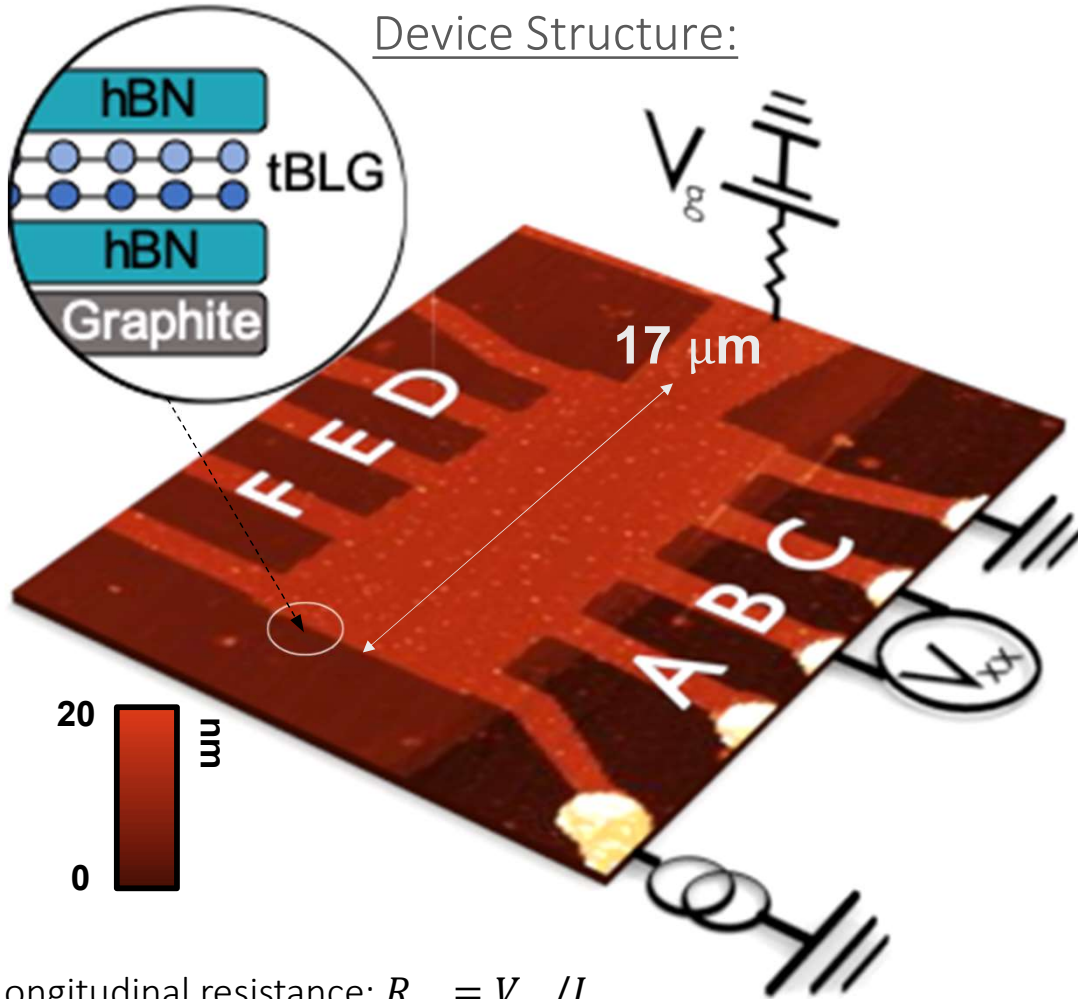


$\lambda \sim 1\mu\text{m}$   
 $U \sim \text{peV}$

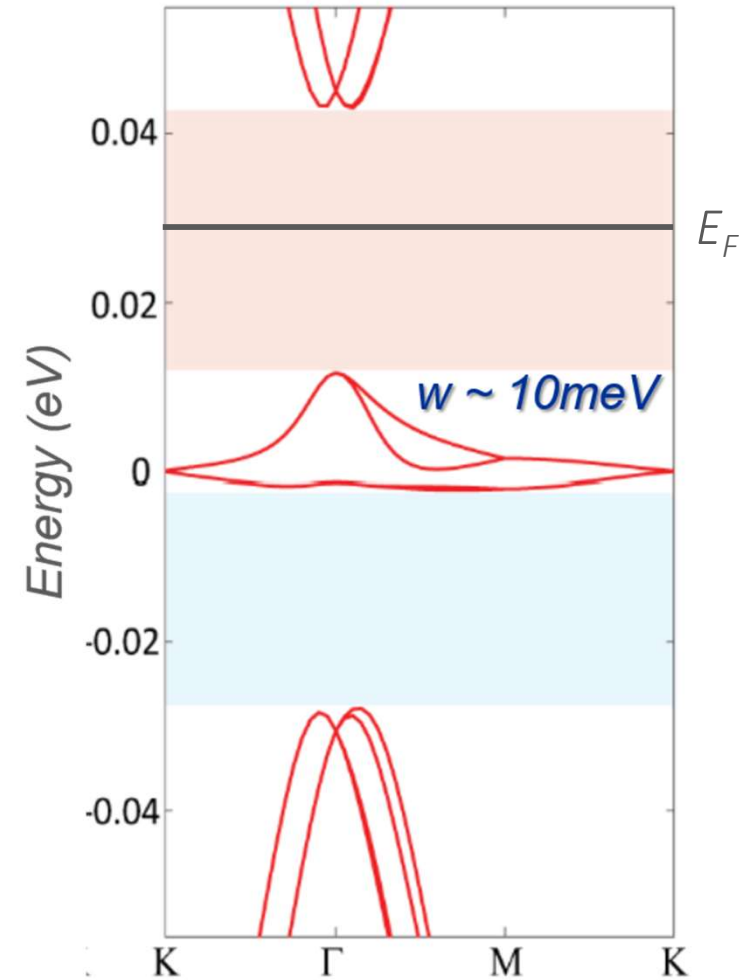
Lattice constant  
Interaction strength

# Transport Measurements

Device Structure:



Tuning Fermi Energy:



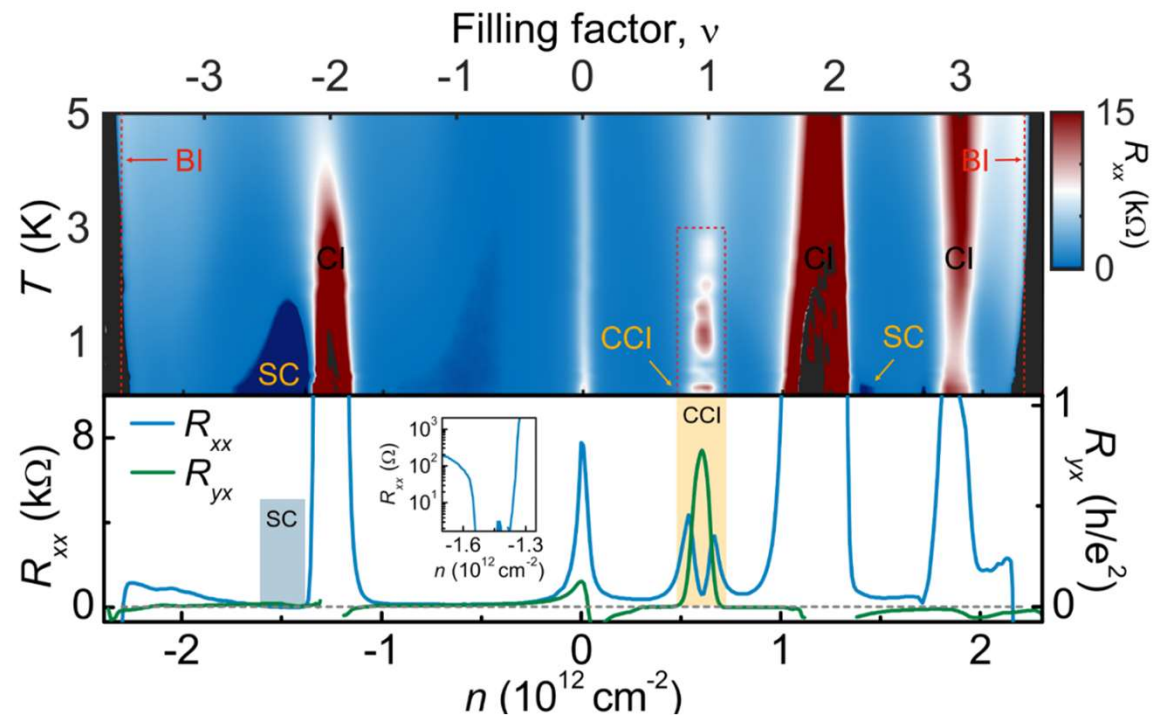
Longitudinal resistance:  $R_{xx} = V_{xx}/I$

Hall resistance:  $R_{xy} = V_{xy}/I$

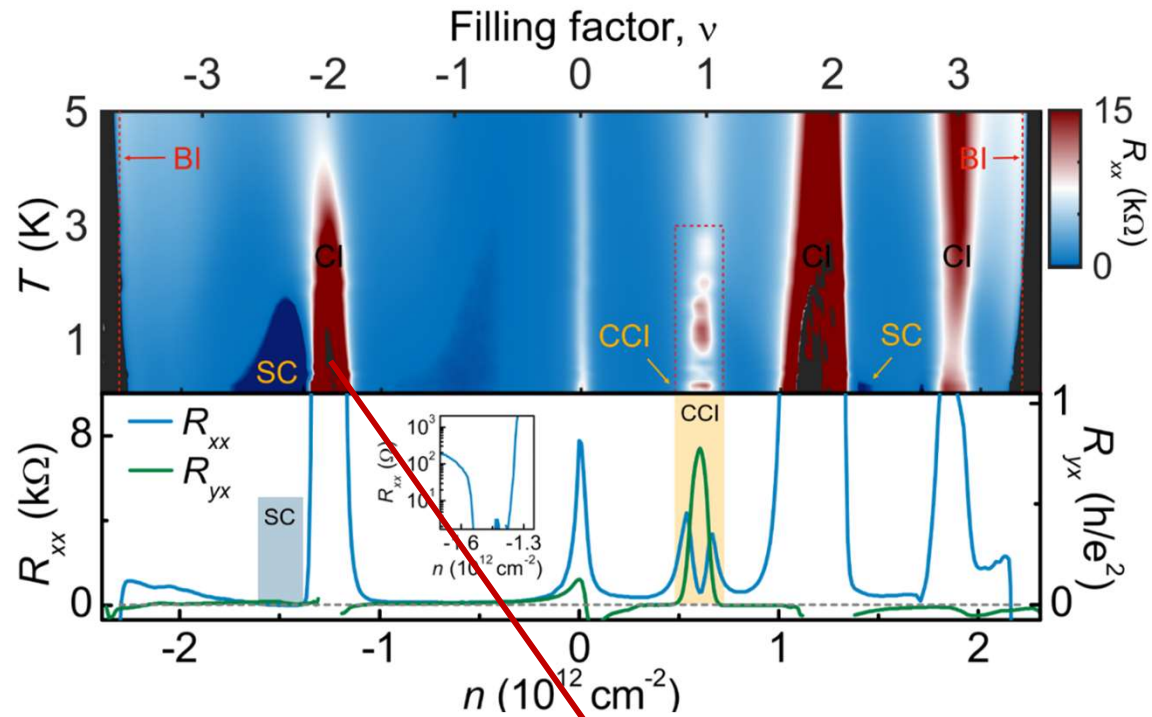
Carrier density:  $n = C_g * V_g$

Hall density:  $n = n_H = -B/eR_{xy}$

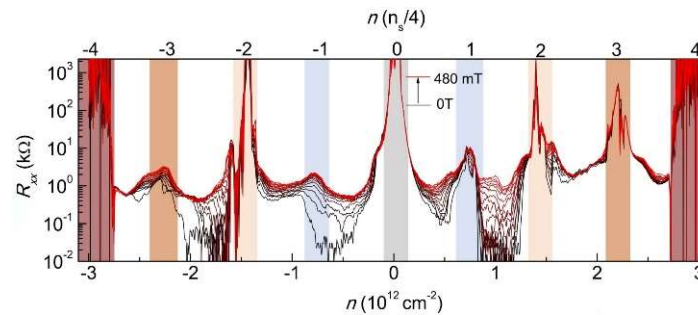
# Magic angle twisted bilayer graphene - Phase diagram



# Magic angle twisted bilayer graphene - Phase diagram

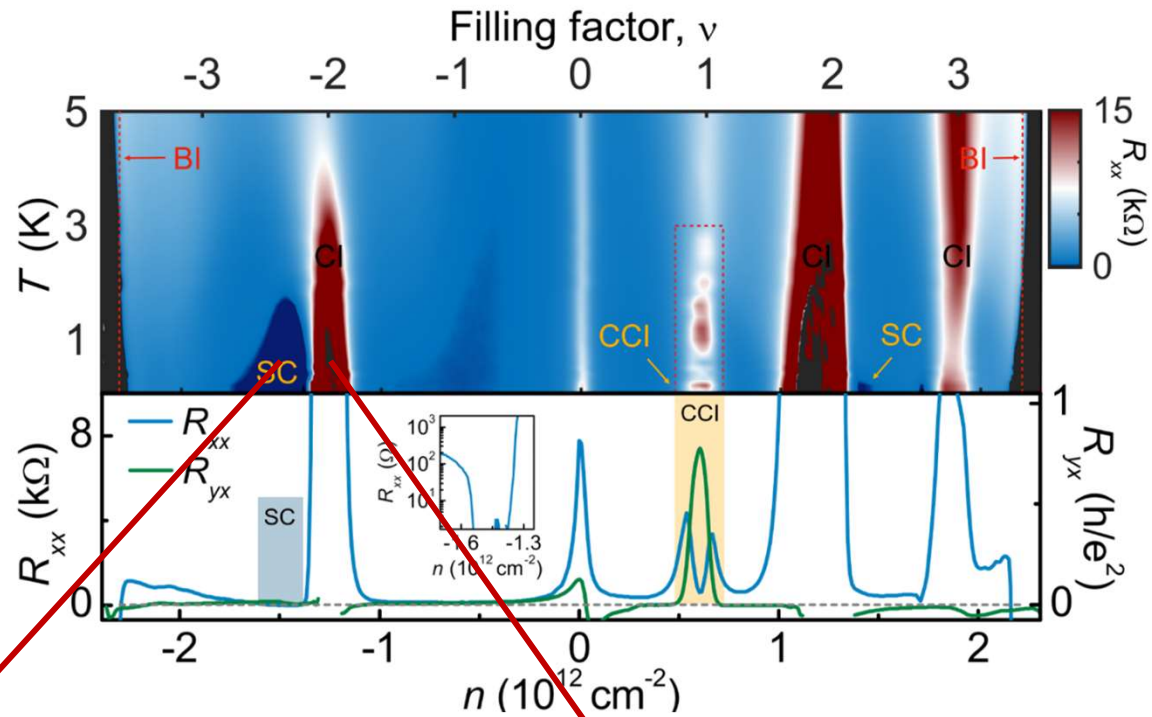


Correlated Insulators:

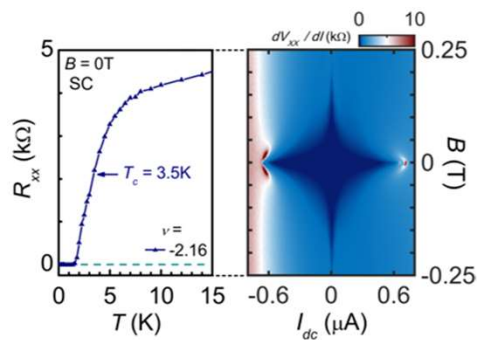


X. Lu, ..., DKE, Nature 574, 653 (2019).

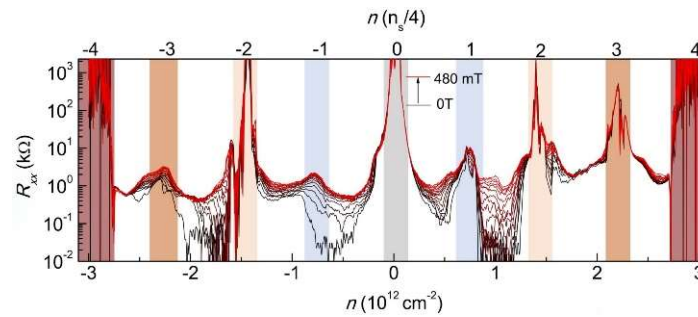
# Magic angle twisted bilayer graphene - Phase diagram



Superconductivity:



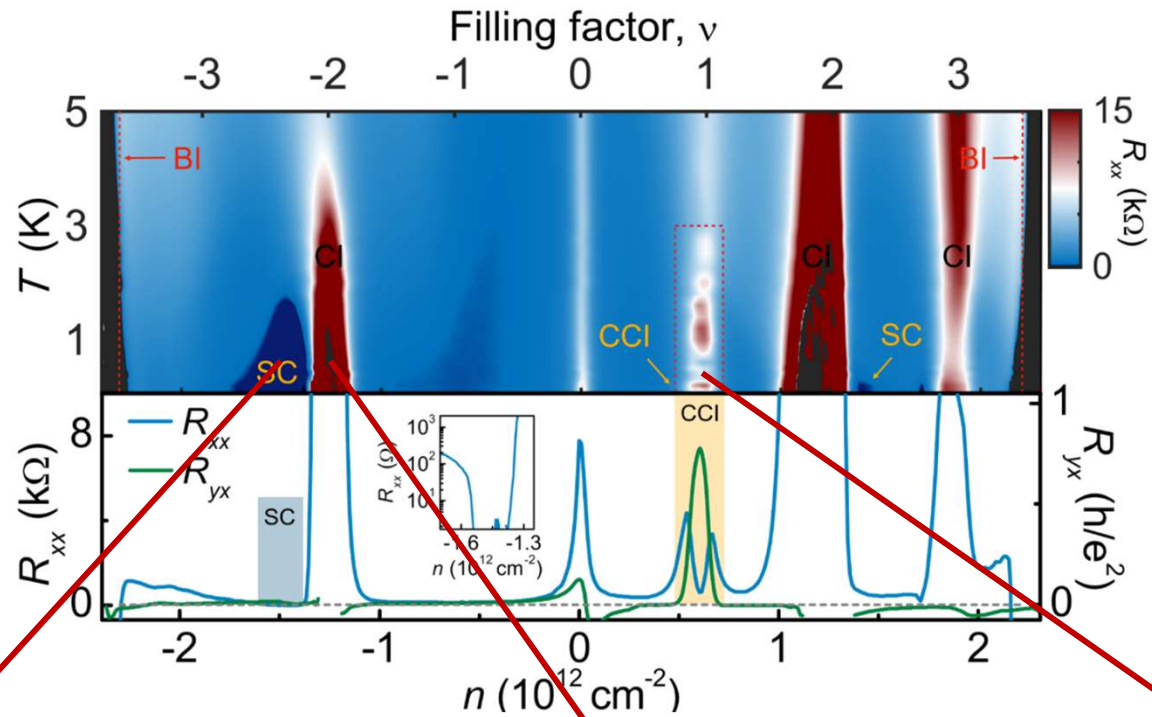
Correlated Insulators:



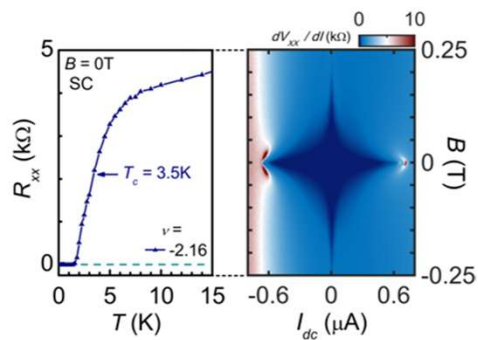
P. Stepanov, I. Das, X. Lu, ... , DKE, Nature 583, 375 (2020).

X. Lu, ... , DKE, Nature 574, 653 (2019).

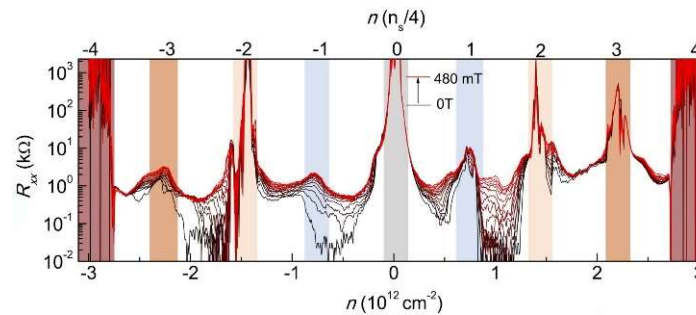
# Magic angle twisted bilayer graphene - Phase diagram



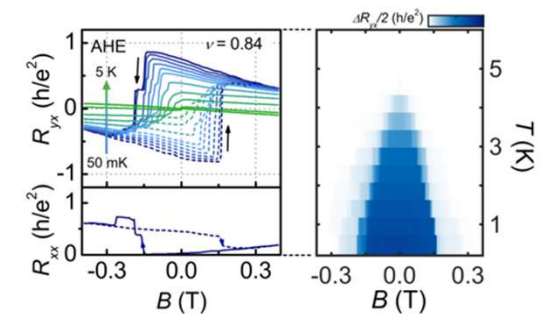
Superconductivity:



Correlated Insulators:



Orbital Magnetism:

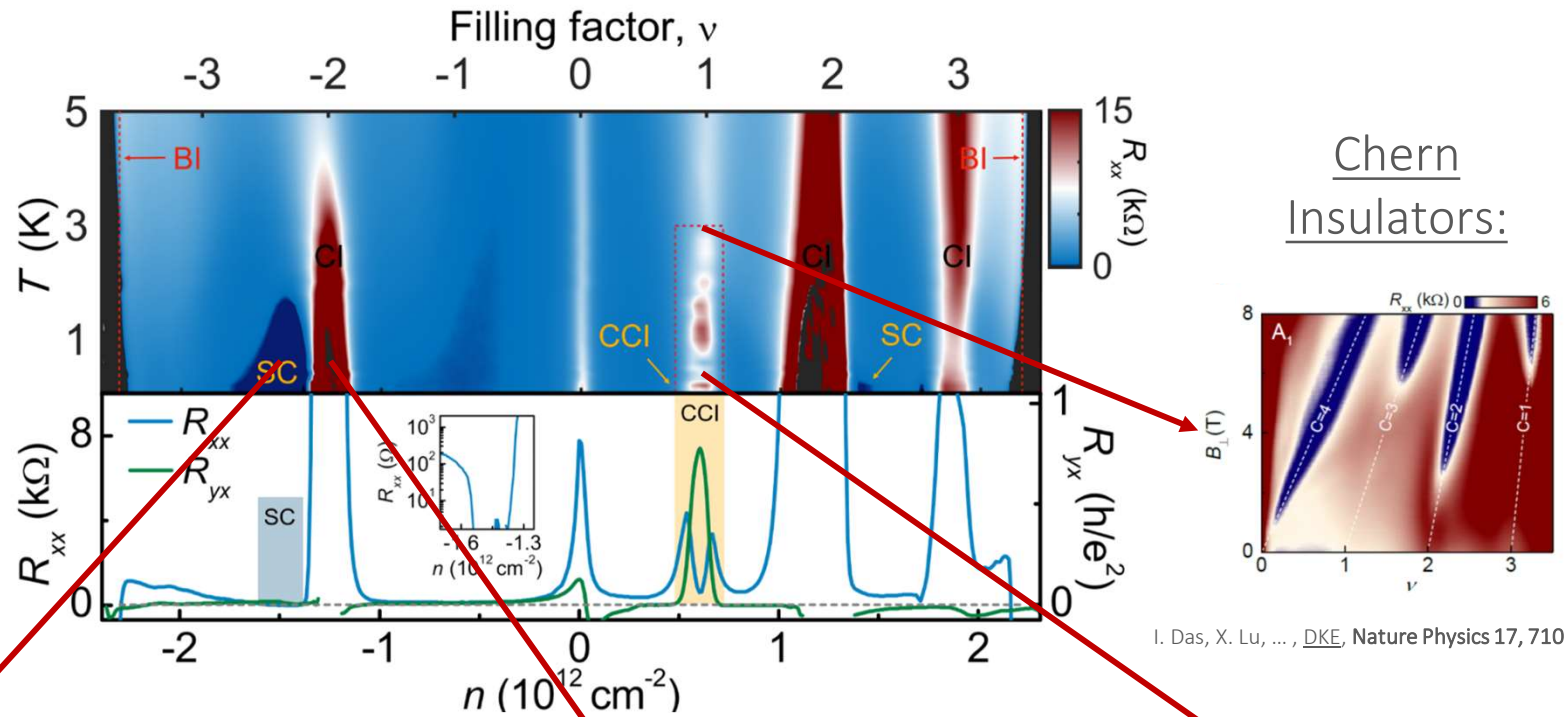


P. Stepanov, I. Das, X. Lu, ... , DKE, Nature 583, 375 (2020).

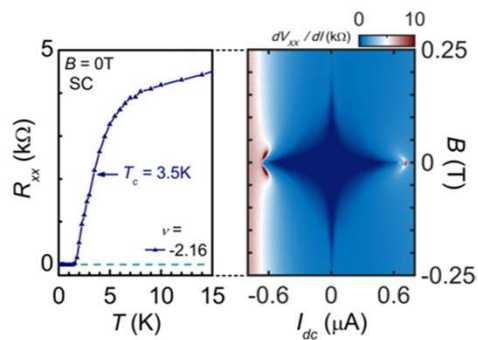
X. Lu, ... , DKE, Nature 574, 653 (2019).

P. Stepanov, ... , A. Bernevig, DKE, PRL 127, 197701 (2021).

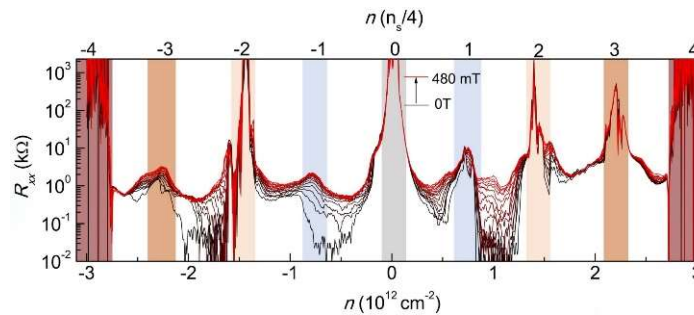
# Magic angle twisted bilayer graphene - Phase diagram



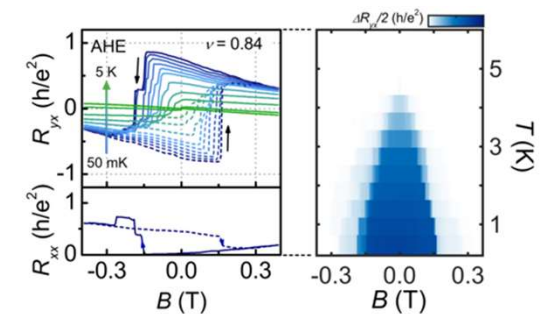
Superconductivity:



Correlated Insulators:



Orbital Magnetism:

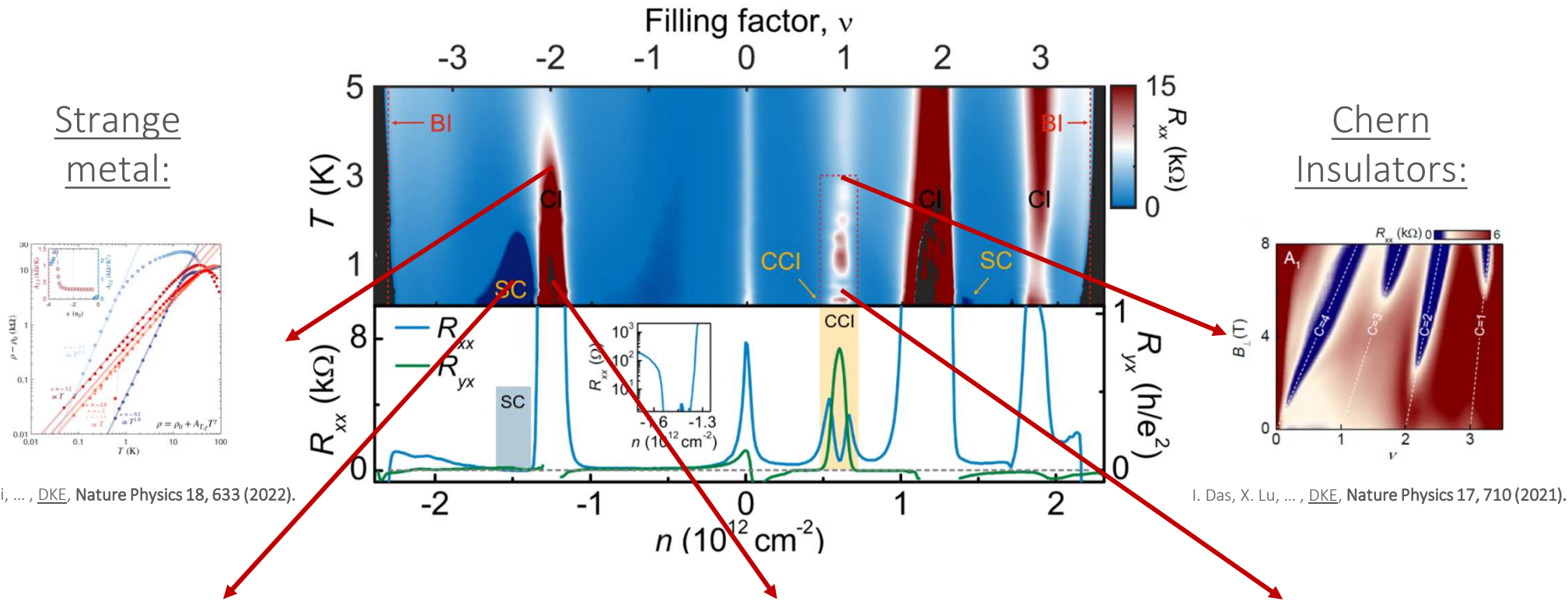


P. Stepanov, I. Das, X. Lu, ... , DKE, Nature 583, 375 (2020).

X. Lu, ... , DKE, Nature 574, 653 (2019).

P. Stepanov, ... , A. Bernevig, DKE, PRL 127, 197701 (2021).

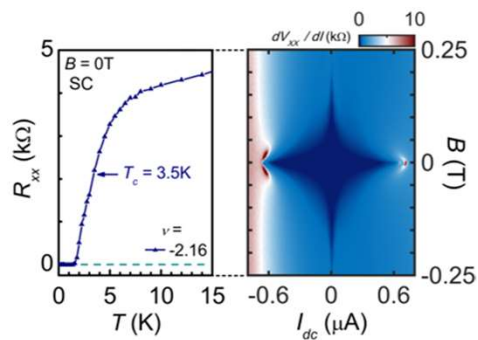
# Magic angle twisted bilayer graphene - Phase diagram



A. Jaoui, ... , DKE, Nature Physics 18, 633 (2022).

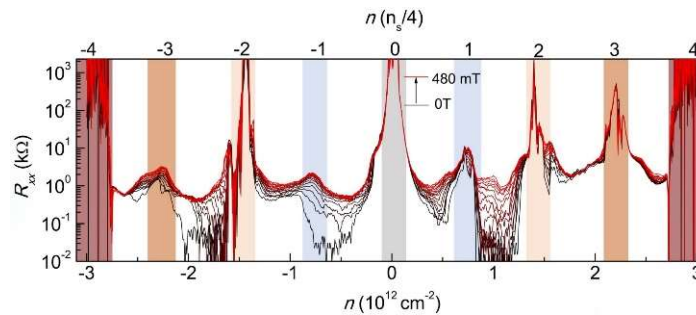
I. Das, X. Lu, ... , DKE, Nature Physics 17, 710 (2021).

Superconductivity:



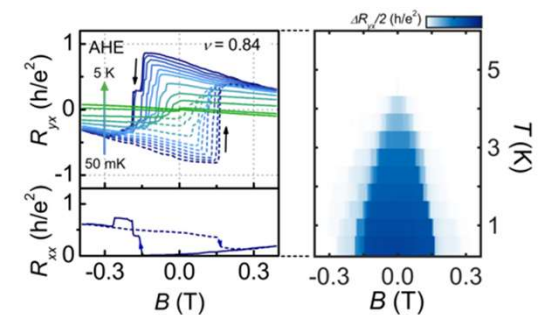
P. Stepanov, I. Das, X. Lu, ... , DKE, Nature 583, 375 (2020).

Correlated Insulators:



X. Lu, ... , DKE, Nature 574, 653 (2019).

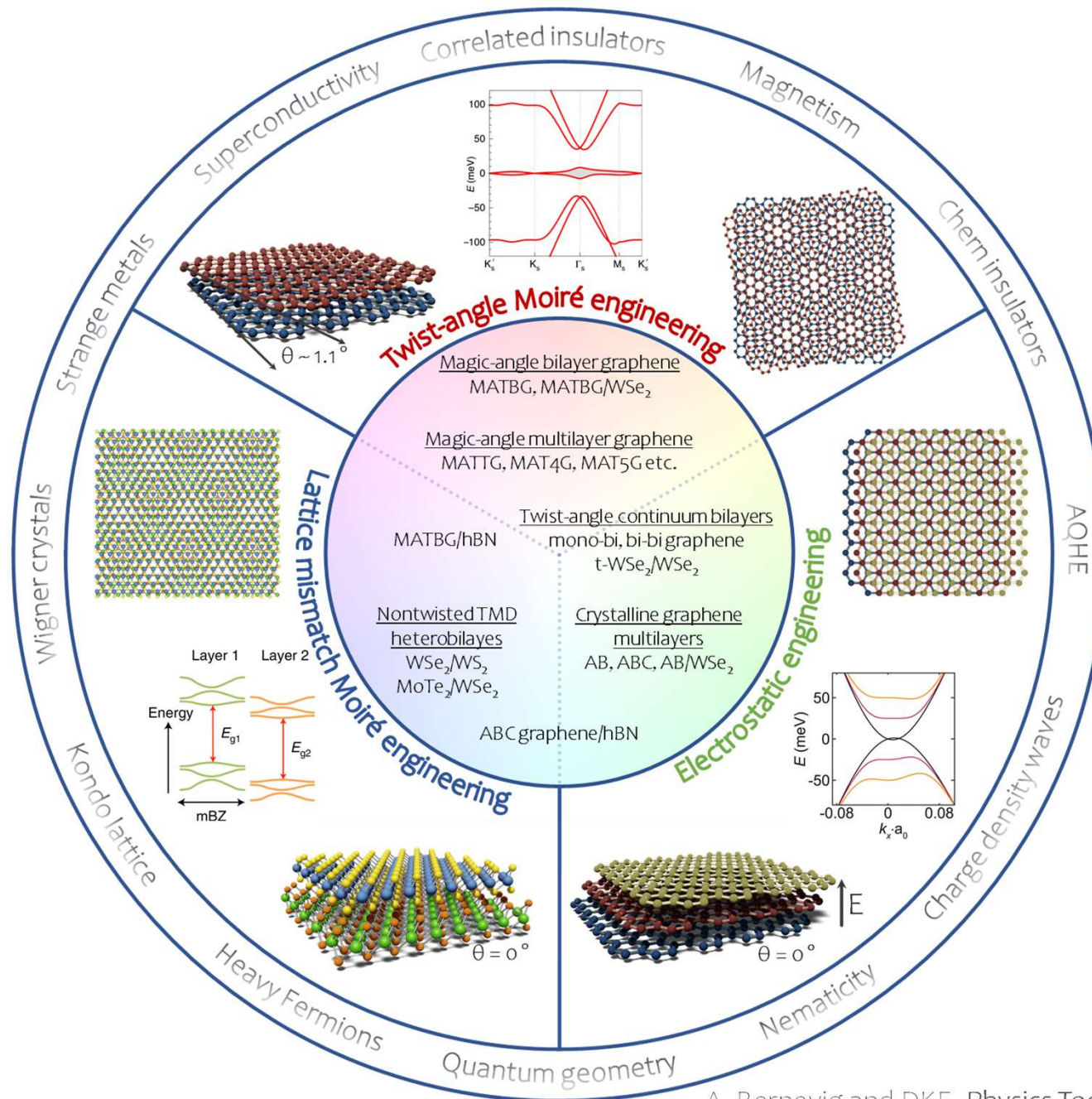
Orbital Magnetism:



P. Stepanov, ... , A. Bernevig, DKE, PRL 127, 197701 (2021).



# Engineering flat bands in 2D materials



A. Bernevig and DKE, Physics Today – in press (2024).