Introduction to graphene and 2D materials

Homework problem set 2

27.05.2024

1 Classical transport

A new semiconducting material has been discovered! As a curious experimental physicist, you went to the cleanroom of the chair of solid state physics to fabricate field effect transistors (FET) made of this new material. The device is made into a transmission line method (TLM) geometry which can be used for both contact resistance measurements and for the characterization of the electronic properties of this material. Figure 1a-b show the device geometry where $w = 2$ μm, $t_{\text{ox}} = 300$ nm, $t = 10$ nm, $d_1 = 1$ μm, $d_2 = 5$ μm, $d_3 = 10$ μm. Using the second and third electrodes (across $d_2 = 5$ μm) as source (s), drain (d) and the highly doped silicon as backgate (bg), you perform the FET measurement shown in Fig. 1c. With all pairs of electrodes, you perform the TLM measurement.

(a) The field effect mobility of thin-films is expressed as $\mu = \frac{dI_{ds}}{dV_{bg}} \frac{d}{wCV_{ds}}$ where d is the length of the channel, w the width of the channel, V_{ds} the applied source-drain voltage, V_{bg} back gate voltage, I_{ds} the source-drain current and C the geometric gate-dielectric capacitance per unit area. What is the highest mobility observed in this device?

(b) All possible configurations for two-probe resistance measurements have been conducted at $V_{\text{bg}} = 2$ V and shown in table 1. Plot the R vs d graph and fit a linear curve from which you should extract the contact resistance, R_c and the resistivity ρ of the material.

(c) What is the charge carrier density of this material at V_{bg} =2 V?

(d) What are the majority carriers in this material? Assuming single particle transport at $V_{bg} = 2$ V, what is the Hall coefficient of this material? Plot the expected classical Hall measurement ρ_{xy} vs B.

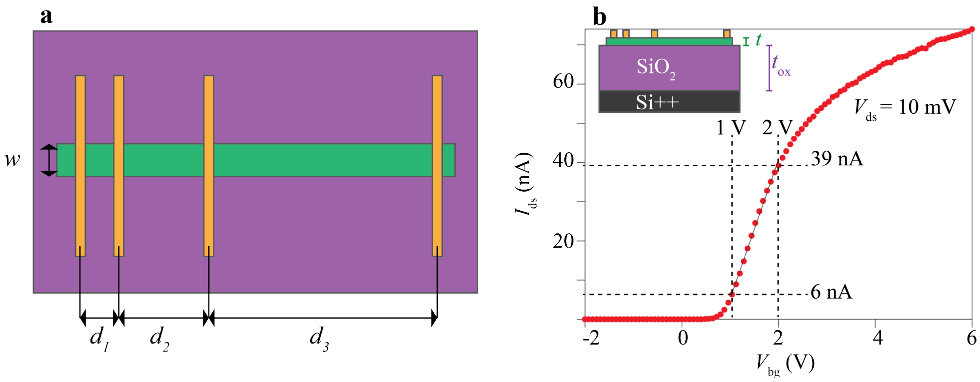

Figure 1: **a** Birds eye view of the device made of 4 electrodes (orange) deposited on the semiconductor (green). **b** FET characteristics measured across the second and third electrode across the channel length d_2. The dotted lines show the region with the highest slope.

d (μm)	R ($k\Omega$)
1	5.12
5	6.73
6	7.09
10	8.68
15	10.71
16	11.22

Table 1: Two-probe resistance values extracted from the TLM measurement.

2 Landau Levels

(a) Consider the Hamiltonian of graphene to be, $H_D = \pm v_F \vec{p} \cdot \vec{\sigma}$ for the K and K' points respectively, where $\vec{p}$ is the momentum and $\vec{\sigma}$ are the spin Pauli matrices (both in the $x - y$ plane). Calculate the energy states of this system for the Landau levels (LL).

(b) What is the degeneracy of each LL? Can you comment on the gap size of these levels compared to the conventional case?

(c) Derive the relation between the LL and the carrier concentration.

3 Dirac representation and Klein tunneling

(a) Since the discovery of Dirac fermions in graphene, many other materials such as topological insulators and topological semimetals have also emerged as systems with Dirac cones. Briefly describe why or why not these systems would facilitate Klein tunneling.

(b) Phosphorene is a single layer of black phosphorus that is known for its highly anisotropic structure (see Fig. 2a). Its structure can be simplified by projecting the puckered hexagonal structure onto the $x - y$ plane as shown in Fig. 2b, where the resulting structure is effectively a distored trigonal lattice with a diatomic (purple and orange) basis. Perform the 2D tight binding calculations and represent the resulting Hamiltonian in terms of Pauli matrices. Show that the dispersion (at energies away from the gap) is linear along one direction while in the other direction, the dispersion is quadratic.

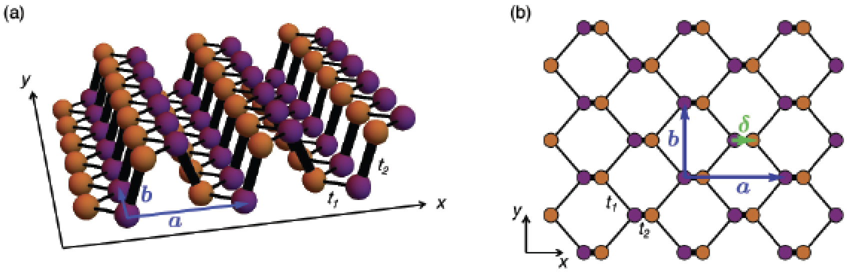

Figure 2: **a** 3D view of the structure of black phosphorous. **b** Orthographic representation of the crystal structure of phosphorene. $t_1 = -1.2$ eV and $t_2 = 3.7$ eV are the hopping parameters.

(c) Bilayer graphene can be described by an effective Hamiltonian $H_2 = -\frac{\hbar}{2m}\begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}$. Show that for a barrier as shown in Fig. 3b, bilayer graphene prohibits Klein tunneling.

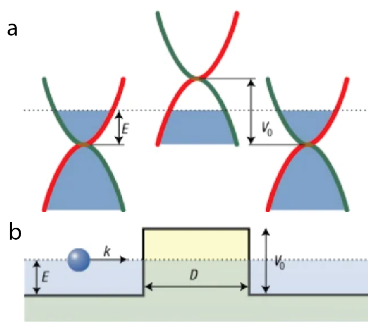

Figure 3: **a** Band structure and energy levels of the npn junction of bilayer graphene. **b** illustration of the potential barrier caused by the npn junction.